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Non-renormalization of the full $\langle VVA \rangle$ correlator at two–loop order

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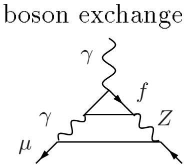
By explicit calculation of the two–loop QCD corrections we show that for singlet axial and vector currents the full off–shell $\langle VVA \rangle$ correlation function in the limit of massless fermions is proportional to the one–loop result, when calculated in the $\overline{\mathrm{MS}}$ scheme. By the same finite renormalization which is needed to make the one–loop anomaly exact to all orders, we arrive at the conclusion that two–loop corrections are absent altogether, for the complete correlator not only its anomalous part. In accordance with the one–loop nature of the $\langle VVA \rangle$ correlator, one possible amplitude, which seems to be missing by accident at the one–loop level, also does not show up at the two–loop level.

1. INTRODUCTION

Recently, Vainshtein [1] found an important new relation between form factors of the $\langle VVA \rangle$ correlator matching to all orders in perturbation theory, in some kinematic limit, a transversal amplitude to the anomalous longitudinal one, which is known to be subject to the Adler-Bardeen non-renormalization theorem [2]. Later Knecht et al. [3] were confirming this kind of non-renormalization theorem. These recent investigations came up in connection with problems in calculating the leading hadronic effects in the electroweak two–loop contributions to the muon anomalous magnetic moment a_μ [4–7].

The diagrams which yield the leading corrections are those including a VVA triangular fermion–loop ($VVA \neq 0$ while $VVV = 0$) associated with a Z boson exchange



and a fermion of flavor f gives a potentially large contribution, up to UV singular terms which will cancel [8],

$$a_\mu^{(4)\,\mathrm{EW}}([f]) \simeq \frac{\sqrt{2} G_\mu m_\mu^2}{16\pi^2} \frac{\alpha}{\pi}\, 2 T_f N_{cf} Q_f^2 \left[3 \ln \frac{M_Z^2}{m_{f'}^2} + C_f \right] \tag{1}$$

where α is the fine structure constant, G_μ the Fermi constant, T_{3f} the 3rd component of the weak isospin, Q_f the charge and N_{cf} the color factor, 1 for leptons, 3 for quarks. The mass $m_{f'}$ is m_μ if $m_f < m_\mu$ and m_f if $m_f > m_\mu$. C_f denotes constant terms. Since, as granted in the Standard Model of elementary particles, anomaly cancellation by lepton–quark duality $\sum_f N_{cf} Q_f^2 T_{3f} = 0$ is at work, only the sums over complete lepton–quark families yield meaningful results relevant to physics. In any case the quark contributions have to be taken into account, and treating them as free fermions the leading large log $\sim \ln M_Z$ drops in sum over each family due to the anomaly cancellation condition of the SM.

However, quarks cannot be treated perturbative and we expect substantial strong interaction effects. A framework to investigate the latter is to consider the general structure of the VVA three point function

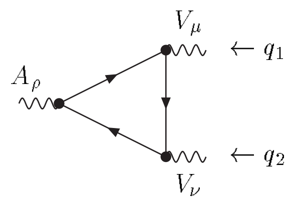
$$\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = i \int d^4x_1 d^4x_2\, e^{i(q_1\cdot x_1 + q_2\cdot x_2)} \times \langle\, 0\, |\, \mathrm{T}\{V_\mu(x_1) V_\nu(x_2) A_\rho(0)\}\, |\, 0\, \rangle \tag{2}$$

of the flavor and color diagonal fermion currents

$$V_\mu = \overline{\psi}\gamma_\mu\,\psi \quad , \quad A_\mu = \overline{\psi}\gamma_\mu\gamma_5\,\psi \tag{3}$$

where ψ is a quark field.

To leading order the correlator of interest is associated with the one–loop triangle diagram



plus its crossed $(q_1, \mu \leftrightarrow q_2, \nu)$ partner.

For the static low energy quantity $a_\mu = \frac{1}{2}(g-2)_\mu = F_{\rm M}(0)$, given by the Pauli form factor at zero momentum transfer, the VVA correlator is required in the limit

$$\begin{aligned}\mathcal{W}_{\mu\nu\rho}(q_1 = k+q, q_2 = -k) \quad &= \qquad (4)\\ -\frac{1}{8\pi^2}\Big\{ w_L\,\left(q^2,0,q^2\right)\, q_\rho\,\varepsilon_{\mu\nu\alpha\beta}\; q^\alpha k^\beta & \\ + w_T\,\left(q^2,0,q^2\right)\, t_{T\;\mu\nu\rho}\Big\} + O(k^2)\,,&\end{aligned}$$

with

$t_{T\;\mu\nu\rho} = \left\{ q^2 \varepsilon_{\mu\nu\rho\sigma} k^\sigma + q_\mu \varepsilon_{\rho\nu\alpha\beta} q^\alpha k^\beta - q_\rho \varepsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta \right\}$.

Indeed, in this kinematic region, the leading strong interaction effects may be parametrized by two VVA amplitudes, a longitudinal $w_L(Q^2)$ and a transversal $w_T(Q^2)$ one as functions of $Q^2 = -q^2$, which contribute as [6, 7]

$$\begin{aligned}\Delta a_\mu^{(4)\,{\rm EW}}([f])_{\rm VVA} \quad &\simeq \quad \frac{\sqrt{2}G_\mu\, m_\mu^2}{16\pi^2}\,\frac{\alpha}{\pi}\,\times \qquad (5)\\ \int_{m_\mu^2}^{\Lambda^2} dQ^2\,&\left(w_L(Q^2) + \frac{M_Z^2}{M_Z^2+Q^2}\, w_T(Q^2)\right)\,,\end{aligned}$$

where Λ is a cutoff to be taken to ∞ at the end. Vainshtein [1] has shown that in the chiral limit the relation

$$w_T(Q^2)_{\rm pQCD}\big|_{m=0} = \frac{1}{2}\;\, w_L(Q^2)\big|_{m=0}\;, \quad (6)$$

which was known to hold at one–loop [9], is valid actually to all orders of perturbative QCD. Vainshtein's theorem follows from the symmetry $(\rho, q \leftrightarrow \mu, q+k)$ $(k \to 0)$. Formally, discarding regularization problems, the asymptotic symmetry derives from the fact that γ_5 may be moved from the A_ρ vertex to the V_μ vertex by anticommuting it an even number of times [1]. Thus for the quarks the non-renormalization theorem valid beyond pQCD for the anomalous amplitude w_L

$$w_L(Q^2)\big|_{m=0} = w_L^{1-{\rm loop}}(Q^2)\Big|_{m=0} = \sum_q \frac{4N_c T_q Q_q}{Q^2}$$

carries over to the perturbative part of the transversal amplitude. Thus in the chiral limit the perturbative QPM result for w_T is exact. This may be somewhat puzzling, since in low energy effective QCD, which encodes the non-perturbative strong interaction effects, this kind of term seems to be absent. The term is recovered however by taking into account all relevant terms in the operator product expansion [3, 7].

In Vainshtein's kinematic limit, what matters is the derivative with respect to k taken at $k = 0$. In this case actually the vertex problem reduces to a propagator type problem. In the calculation described below we have extended this to a genuine vertex type statement at the two–loop level. As the extensions of the Adler-Bardeen non-renormalization theorem for the anomalous Ward identity $\langle VV\partial A\rangle$ turn out to play an important role in new phenomenological applications, we will study in the following such possible generalizations by an explicit calculation of the leading QCD corrections to the $\gamma\gamma Z$ triangle.

The vector currents are strictly conserved $\partial_\mu V^\mu = 0$, while the axial vector current satisfies a PCAC relation plus the anomaly $\partial_\mu A^\mu = 2im_0\bar{\psi}\gamma_5\psi + \frac{\alpha_0}{4\pi}\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu}(x)F^{\rho\sigma}(x)$. We will be mainly interested in the properties of strongly interacting quark flavor currents in perturbative QCD. Our notation closely follows [3].

The Ward identities restrict the general covariant decomposition of $\mathcal{W}_{\mu\nu\rho}(q_1, q_2)$ into invariant functions to four terms

$$\begin{aligned}-\,8\pi^2\,\mathcal{W}_{\mu\nu\rho}(q_1,q_2) \quad &= \\ & w_L\,\left(q_1^2,q_2^2,q_3^2\right)\, q_{3\rho}\,\varepsilon_{\mu\nu\alpha\beta}\; q_1^\alpha q_2^\beta \\ & + w_T^{(+)}\,\left(q_1^2,q_2^2,q_3^2\right)\, t^{(+)}_{\mu\nu\rho}(q_1,q_2) \\ & + w_T^{(-)}\,\left(q_1^2,q_2^2,q_3^2\right)\, t^{(-)}_{\mu\nu\rho}(q_1,q_2) \\ & + \widetilde{w}_T^{(-)}\,\left(q_1^2,q_2^2,q_3^2\right)\, \widetilde{t}^{(-)}_{\mu\nu\rho}(q_1,q_2)\,, \quad (7)\end{aligned}$$

with $-q_3 = q_1 + q_2$ and transverse tensors given by

$$t^{(+)}_{\mu\nu\rho}(q_1,q_2) = q_{1\nu}\,\varepsilon_{\mu\rho\alpha\beta}\; q_1^\alpha q_2^\beta - q_{2\mu}\,\varepsilon_{\nu\rho\alpha\beta}\; q_1^\alpha q_2^\beta$$
$$- q_1 q_2\,\varepsilon_{\mu\nu\rho\alpha}\;(q_1-q_2)^\alpha + \frac{2\,q_1 q_2}{q_3^2}\;\varepsilon_{\mu\nu\alpha\beta}\; q_1^\alpha q_2^\beta q_{3\rho}$$
$$t^{(-)}_{\mu\nu\rho}(q_1,q_2) =$$
$$\left[(q_1-q_2)_\rho + \frac{q_1^2-q_2^2}{q_3^2}\,q_{3\rho}\right]\varepsilon_{\mu\nu\alpha\beta}\; q_1^\alpha q_2^\beta$$
$$\widetilde{t}^{(-)}_{\mu\nu\rho}(q_1,q_2) = q_{1\nu}\,\varepsilon_{\mu\rho\alpha\beta}\; q_1^\alpha q_2^\beta$$
$$+ q_{2\mu}\,\varepsilon_{\nu\rho\alpha\beta}\; q_1^\alpha q_2^\beta + q_1 q_2\,\varepsilon_{\mu\nu\rho\alpha}\; q_3^\alpha\,.$$

The longitudinal part is entirely fixed by the anomaly,

$$w_L\left(q_1^2,q_2^2,q_3^2\right) = -\frac{2N_c}{q_3^2} \tag{8}$$

which is exact to all orders of perturbation theory, the famous Adler-Bardeen non-renormalization theorem. The Vainshtein relation is obtained in the limit (4) upon identifying

$$w_L(Q^2) \equiv w_L(q^2,0,q^2)\,,$$
$$w_T(Q^2) \equiv w_T^{(+)}(q^2,0,q^2) + \widetilde{w}_T^{(-)}(q^2,0,q^2)\,,$$

with $Q^2=-q^2$.

2. THE CALCULATION

Here we report on recent progress we made in extending non-renormalization phenomenon at the two–loop level [10] . For details and further references we refer to the latter paper in the following. We perform the calculation with conventional dimensional regularization in $d=4-2\varepsilon$ dimensions and use a linear covariant gauge with arbitrary gauge parameter throughout the calculation. Our procedure of treating γ_5 is similar to the one used in [11]. We write down all fermion loops starting with the axial-vector vertex, and then perform Feynman integrals and Dirac algebra without assuming any property of γ_5 at all. In this way all diagrams will be expressed in terms of traces of 10 combinations of γ matrices. The prescription is sufficient to enable us to arrive at amplitudes which have finite limits as $d\to 4$ in the corresponding covariant decomposition. After this the usual formulas

$$\mathrm{Tr}[\gamma_5\gamma_\alpha\gamma_\beta\gamma_\mu\gamma_\nu] = 4i\varepsilon_{\alpha\beta\mu\nu}\ ,\ \ \mathrm{Tr}[\gamma_5\gamma_\alpha\gamma_\beta]=0$$

valid in $d=4$ dimensions were used. In our convention $\varepsilon_{0123}=+1$ and $(1-\gamma_5)/2$ projects to left–handed fermion fields.

Tensor integrals were expressed in terms of integrals with different shifts of the space-time dimension [12]. All scalar integrals could be reduced to 6 master integrals by using the Gröbner basis technique proposed in [13]. The expressions for the individual diagrams are sums over 21 terms which are combinations of the 6 master integrals

$$I_2^{(d)}(q_1^2) = \int\frac{\widetilde{d^dk_1}}{D_1D_3}, \qquad \widetilde{d^dk_j} = \frac{d^dk_j}{i\pi^{d/2}},$$
$$I_3^{(d)}(q_1^2,q_2^2,q_3^2) = \int\frac{\widetilde{d^dk_1}}{D_1D_3D_4}$$
$$J_3^{(d)}(q_1^2) = \int\int\frac{\widetilde{d^dk_1}\widetilde{d^dk_2}}{D_1D_5D_6},$$
$$R_1(q_1^2,q_2^2,q_3^2) = \int\int\frac{\widetilde{d^dk_1}\widetilde{d^dk_2}}{D_1D_5D_6D_7}$$
$$R_2(q_1^2,q_2^2,q_3^2) = \int\int\frac{\widetilde{d^dk_1}\widetilde{d^dk_2}}{D_1^2D_5D_6D_7},$$
$$P_5(q_1^2,q_2^2,q_3^2) = \int\int\frac{\widetilde{d^dk_1}\widetilde{d^dk_2}}{D_1D_2D_5D_3D_7}, \tag{9}$$

multiplied by ratios of polynomials in q_j^2 and d. Here $D_1=k_1^2$, $D_2=k_2^2$, $D_3=(k_1-q_1)^2$, $D_4=(k_1+q_2)^2$, $D_5=(k_1-k_2)^2$, $D_6=(k_2-q_1)^2$ and $D_7=(k_2+q_2)^2$. The integrals (9) form a complete set of master integrals needed for the calculation of massless vertex diagrams with planar topology. The planar integral with 6 denominators

$$P_6(q_1^2,q_2^2,q_3^2) = \int\int\frac{\widetilde{d^dk_1}\widetilde{d^dk_2}}{D_1D_3D_4D_5D_6D_7}$$

can be reduced to integrals (9) using

$$q_3^2\varepsilon P_6 = (1-2\varepsilon)I_3(q_2^2,q_3^2,q_1^2)I_2^{(d)}(q_3^2)$$
$$-R_2(q_1^2,q_2^2,q_3^2) + R_2(q_1^2,q_3^2,q_2^2) + R_2(q_2^2,q_3^2,q_1^2)$$
$$+\varepsilon(P_5(q_1^2,q_3^2,q_2^2) + P_5(q_2^2,q_3^2,q_1^2)).$$

R_1 satisfies the system of differential equations:

$$\{x(1-x)\partial_x^2 - y^2\partial_y^2 + [\gamma - (\alpha+\beta+1)x]\partial_x$$
$$-2xy\partial_x\partial_y - (\alpha+\beta+1)y\partial_y - \alpha\beta\}R_1 = 0,$$
$$\{y(1-y)\partial_y^2 - x^2\partial_x^2 + [\gamma - (\alpha+\beta+1)y]\partial_y$$
$$-2xy\partial_x\partial_y - (\alpha+\beta+1)x\partial_x - \alpha\beta\}R_1 = 0,$$

where $\beta = \gamma = 2\alpha = 2\varepsilon$ and

$$x = \frac{q_1^2}{q_3^2}, \quad y = \frac{q_2^2}{q_3^2}, \quad \partial_x = \frac{\partial}{\partial x}, \quad \partial_y = \frac{\partial}{\partial y}.$$

The general solution of this system may be written in terms of Appell functions F_4

$$\begin{aligned}(-q_3^2)^{2\varepsilon}R_1 = {} & AF_4(\varepsilon, 2\varepsilon, 2\varepsilon, 2\varepsilon; x, y)\\ & +BF_4(1-\varepsilon, 1, 2-2\varepsilon, 2\varepsilon; x, y)x^{1-2\varepsilon}\\ & +CF_4(1-\varepsilon, 1, 2\varepsilon, 2-2\varepsilon; x, y)y^{1-2\varepsilon}\\ & +DF_4(2-3\varepsilon, 2-2\varepsilon, 2-2\varepsilon, 2-2\varepsilon; x, y),\end{aligned}$$

where

$$A = \frac{\Gamma(2\varepsilon)\Gamma^2(1-\varepsilon)\Gamma(\varepsilon)\Gamma(1-2\varepsilon)}{(1-2\varepsilon)\Gamma(2-3\varepsilon)},$$
$$B = C = \frac{\Gamma(-1+2\varepsilon)\Gamma^3(1-\varepsilon)}{(1-2\varepsilon)\Gamma(2-3\varepsilon)},$$
$$D = \Gamma^2(1-\varepsilon)\Gamma^2(-1+2\varepsilon)x^{1-2\varepsilon}y^{1-2\varepsilon}.$$

The F_4 functions can be simplified yielding

$$\begin{aligned}(-q_3^2)^{2\varepsilon}R_1 = {} & \frac{\Gamma(2\varepsilon)\Gamma^3(1-\varepsilon)}{\varepsilon(1-2\varepsilon)\Gamma(2-3\varepsilon)}\\ & \times\left[\frac{(Q_1+\lambda)}{2yx^{2\varepsilon-1}}F\left(\frac{Q_1+\lambda}{2y}, \frac{Q_1+\lambda}{Q_1-\lambda}\right)\right.\\ & \left.+\frac{(Q_2+\lambda)}{2xy^{2\varepsilon-1}}F\left(\frac{Q_2+\lambda}{2x}, \frac{Q_2+\lambda}{Q_2-\lambda}\right)\right]\\ & +\frac{\pi\Gamma^2\left(\varepsilon-\frac{1}{2}\right)}{16^{1-\varepsilon}\sin(\pi\varepsilon)^2}\left[M\ G\left(\frac{Q_3+\lambda}{2\lambda}\right)\right.\\ & \left.+\frac{2^{1+\varepsilon}\cos(\pi\varepsilon)}{(\lambda-Q_3)^\varepsilon}G\left(\frac{2\lambda}{\lambda-Q_3}\right)\right] \qquad (10)\end{aligned}$$

where $\lambda = \sqrt{\Delta}$ and

$$\Delta = 1 + x^2 + y^2 - 2xy - 2x - 2y,$$
$$Q_1 = y+1-x, \ Q_2 = x+1-y, \ Q_3 = x+y-1,$$

$$G(z) = {}_2F_1\left(\varepsilon, 1-\varepsilon, 2-2\varepsilon, z\right),$$
$$F(z,\omega) = F_1\left(1, 1-\varepsilon, 1-\varepsilon, 1+\varepsilon; z, \omega\right),$$

$$\begin{aligned}M = {} & \left(\frac{Q_3+\lambda}{-2\lambda}\right)^{1-2\varepsilon}\frac{1}{\lambda^\varepsilon}\\ & +\frac{x^{1-2\varepsilon}y^{1-2\varepsilon}}{\lambda^{1-\varepsilon}}\left(\frac{Q_3+\lambda}{-2xy}\right)^{1-2\varepsilon}\\ & -\frac{(1-4\cos(\pi\varepsilon)^2)}{\lambda^{1-\varepsilon}}\left[\left(\frac{Q_3+\lambda}{2x}\right)^{1-2\varepsilon}x^{1-2\varepsilon}\right.\\ & \left.+\left(\frac{Q_3+\lambda}{2y}\right)^{1-2\varepsilon}y^{1-2\varepsilon}\right]. \qquad (11)\end{aligned}$$

The Gauss' hypergeometric function has an integral representation

$$G(z) = \frac{\Gamma(2-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\int_0^1 \frac{du}{[u(1-u)(1-zu)]^\varepsilon} \quad (12)$$

which is convenient for performing the ε expansion. For the expansion of $F(x,y)$ the relation

$$\begin{aligned}&(1+\varepsilon)(1-y)(1-x)(x-y)F(x,y) =\\ &\quad (x-y)[1+(1-x)(1-y)\varepsilon]\phi(x,y)\\ &\quad + x(x-y-x^2y+x^2)\partial_x\phi(x,y)\\ &\quad + y(x-y-y^2+xy^2)\partial_y\phi(x,y), \qquad (13)\end{aligned}$$

may be used to express this function in terms of another F_1 function which is more suitable for ε expansion

$$\begin{aligned}\phi(x,y) = F_1\left(1, -\varepsilon, -\varepsilon, 2+\varepsilon; x, y\right) =\\ (1+\varepsilon)\int_0^1 du[(1-u)(1-xu)(1-yu)]^\varepsilon. \quad (14)\end{aligned}$$

An expression for R_2 may be obtained by differentiating the one given for R_1. The hypergeometric representation for P_5 is obtained by solving the first order difference equation with respect to d. Details of these calculations will be given in a separate publication. Series expansion in ε for various master integrals to the order needed in our calculations was given in [14, 15]. Recently, further terms of the ε expansion for these master integrals were calculated in [16].

The sum of all diagrams turns out to be gauge parameter independent. In the Feynman gauge at $q_3 = 0$ and for arbitrary d, the results of our calculation are in agreement with the ones presented in [11] diagram by diagram.

By applying the prescription outlined above for the evaluation of individual diagrams, and using Schouten's identities, we are able to reshuffle terms to match the tensor structures introduced in (7), exhibiting manifestly the vector current conservation.

3. RESULTS AND DISCUSSION

Including one– and two–loop contributions, we may represent the form-factors in the form

$$\begin{aligned} &w_T^{(\pm)}(q_1^2,q_2^2,q_3^2) = n_f\ N_c\ w_{1,T}^{(\pm)}(q_1^2,q_2^2,q_3^2) \\ &+ a\ n_f\ N_c\ C_2(R)\ w_{2,T}^{(\pm)}(q_1^2,q_2^2,q_3^2) \\ &w_L(q_1^2,q_2^2,q_3^2) = n_f\ N_c\ w_{1,L}(q_1^2,q_2^2,q_3^2) \\ &+ a\ n_f\ N_c\ C_2(R)\ w_{2,L}(q_1^2,q_2^2,q_3^2) \qquad (15) \end{aligned}$$

where $a = \alpha_s/(4\pi)$ includes the QCD coupling α_s, n_f is the number of flavors and N_c the number of colors. The quarks are in the fundamental representation R and the corresponding group theory factor is given by

$$C_2(R) I = R^a R^a\ ,\quad C_2(R) = 4/3 \text{ for QCD}\ . \qquad (16)$$

We have been working in the $\overline{\rm MS}$ renormalization scheme. The singlet axial current $J_\rho^5 = A_\rho$ is non-trivially renormalized because of the axial anomaly. It is known [17] that in addition to the standard ultraviolet renormalization constant $Z_{\overline{\rm MS}}$ which reads $Z_{\overline{\rm MS}} = 1$ in our case (as $Z_{\overline{\rm MS}} - 1 = O(a^2)$), one has to apply a finite renormalization constant Z_5 such that renormalized and bare currents are related as:

$$(J_\rho^5)_r = Z_5 Z_{\overline{\rm MS}}\,(J_\rho^5)_0. \qquad (17)$$

The counterterms coming from the wave function renormalization of quarks and ultraviolet renormalization of the axial and vector currents cancel. The finite renormalization constant is known at the three-loop level [18]. For our calculations we must take

$$Z_5 = 1 - 4C_2(R)\,a\ . \qquad (18)$$

The result of the two–loop calculation after adding all diagrams is surprisingly simple and, normalized according to (15), is given by

$$\begin{aligned} &q_3^2 w_{2,L}(q_1^2,q_2^2,q_3^2) = -8, \\ &\widetilde{w}_{2,T}^{(-)}(q_1^2,q_2^2,q_3^2) = -w_{2,T}^{(-)}(q_1^2,q_2^2,q_3^2), \qquad (19) \\ &q_3^2\Delta^2 w_{2,T}^{(-)}(q_1^2,q_2^2,q_3^2) = 8(x-y)\Delta \\ &\quad + 8(x-y)(6xy+\Delta)\Phi^{(1)}(x,y) \\ &\quad - 4[18xy + 6x^2 - 6x + (1+x+y)\Delta)]L_x \\ &\quad + 4[18xy + 6y^2 - 6y + (1+x+y)\Delta)]L_y \end{aligned}$$

$$\begin{aligned} &q_3^2\Delta^2 w_{2,T}^{(+)}(q_1^2,q_2^2,q_3^2) = +8\Delta \\ &\quad + 8[6xy + (x+y)\Delta]\Phi^{(1)}(x,y) \\ &\quad - 4[6x+\Delta](x-y-1)L_x \\ &\quad + 4[6y+\Delta](x-y+1)L_y \qquad (20) \end{aligned}$$

with $L_x = \ln x, \quad L_y = \ln y$. Expression for $\Phi^{(1)}$ may be found in [19]:

$$\begin{aligned} \Phi^{(1)}(x,y) = \frac{1}{\lambda}\Big\{ &2\left(\mathrm{Li}_2\left(-\rho x\right) + \mathrm{Li}_2\left(-\rho y\right)\right) \\ &+ \ln\frac{y}{x}\ln\frac{1+\rho y}{1+\rho x} + \ln(\rho x)\ln(\rho y) + \frac{\pi^2}{3}\Big\}, \end{aligned} \qquad (21)$$

where

$$\rho(x,y) \equiv 2/(1-x-y+\lambda),$$

The comparison with the results of the one-loop calculation reveals that

$$\begin{aligned} &\mathcal{W}_{\mu\nu\rho}(q_1,q_2)\,|_{two-loop} = \\ &4C_2(R)a\mathcal{W}_{\mu\nu\rho}(q_1,q_2)\,|_{one-loop} \end{aligned} \qquad (22)$$

Multiplying the sum of one- and two-loop terms by the finite factor Z_5 we arrive at

$$\mathcal{W}_{\mu\nu\rho}(q_1,q_2) = \mathcal{W}_{\mu\nu\rho}(q_1,q_2)\,|_{one-loop} \qquad (23)$$

This is the non-renormalization theorem for the full off shell correlator at two–loops. While our calculation confirms the relations derived in [3] and the non-renormalization theorem (6) found in [1,7], these findings are not sufficient to explain our result valid for generic momenta.

Taking into account the rather non-trivial momentum dependence of the form-factors it is very tempting to suggest that it could hold to all orders of perturbation theory because of the topological nature of the anomaly, for example.

We would like to stress that the surprising relation could be discovered only by keeping the general non-trivial momentum dependence. The anomalous three point correlator exhibits an unusually simple structure, while contributions from individual diagrams are very unwieldy. One can expect similar effects for other anomalous correlators. Since at the order considered the QCD calculation is essentially a QED calculation, it is highly non-trivial whether this carries over to higher orders. For the $\langle VV\partial A\rangle$ anomalous correlator a large number of two–loop calculations have been performed, mainly in QED and we refer to the comprehensive review by Adler [20] and references therein.

In electroweak SM calculations one would a priori expect that renormalizing parameters and fields would be sufficient for renormalizing the SM. Our calculation shows that on top of the standard renormalization, it is mandatory to renormalize the anomalous currents J_ρ^5 by the finite renormalization factor Z_5 because the lepton currents and the quark currents pick different Z–factors and if they are not renormalized away the anomaly cancellation and hence renormalizability obviously would get spoiled. As pointed out by Adler and many others [20] the point is that there exists a renormalization scheme for which the one–loop anomaly is exact. Only in this scheme anomaly cancellation and thus renormalizability will carry over to higher orders in the SM. Our result shows that due to the necessity of renormalizing away possible higher order contributions from the anomaly also the non-anomalous transversal contributions are affected. We have shown that at least at two–loops the entire contribution gets renormalized away in the zero mass limit.

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