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Generalized Multiresolution Analysis on Unstructured Grids

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Abstract

Efficiency of high-order essentially non-oscillatory (ENO) approximations of conservation laws can be drastically improved if ideas of multiresolution analysis are taken into account. These methods of data compression not only reduce the necessary amount of discrete data but can also serve as tools in detecting local low-dimensional features in the numerical solution. We describe the mathematical background of the generalised multiresolution analysis as developed by Abgrall and Harten in [14], [15] and [3]. The functional analytic background is ultimately reduced to matrix-vector operations of linear algebra. We consider the example of interpolation on the line as well as the important case of multiresolution analysis of cell average data which is used in finite volume approximations.

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1 Introduction

One of the most important tasks of Computational Fluid Dynamics (CFD) is the design of highly accurate, robust and efficient numerical methods for the simulation of compressible fluid flow. It is common knowledge in numerical analysis that the three properties: accuracy, robustness, and efficiency, are orthogonal concepts in almost all areas of algorithmic development and that problems due to the complementary nature of these properties can only be circumvented if additional information about the problem to be solved is available. A classical paradigm can be found in the well-known multigrid methods, see [6]. Here, accurate and robust finite element and finite difference methods for elliptic partial differential equations can be made efficient by analyzing the damping properties of linear systems solvers on a sequence of grids with decreasing resolution.
The most sophisticated numerical methods combining accuracy with robustness in the computation of compressible flow fields are finite volume approximations based on essentially non-oscillatory (ENO) recovery procedures, see [2], [8], [23]. While the choice of the type of the recovery function is responsible for high accuracy the choice of the stencil to compute the recovery function adds the required robustness. However, these methods are *a priori* not very efficient since all of the flow field is treated with the same expensive algorithm: stencil selection, ENO recovery, solution of local Riemann problems; but the full ENO algorithm is only required across and close to shocks and contact discontinuities which form lower dimensional manifolds in the solution. Would it be known in advance in which parts of the flow field the discontinuities are located and which parts are dominated by smoothly