A TWO–DIMENSIONAL ELECTRON–HOLE SYSTEM UNDER THE INFLUENCE OF THE CHERN–SIMONS GAUGE FIELD CREATED BY QUANTUM POINT VORTICES

Sveatoslav A. Moskalenko[1], Vsevolod A. Moskalenko[1], Igor V. Podlesny[1], and Michael A. Liberman[2]

[1]*Institute of Applied Physics, Academiei str.5, Chisinau, MD-2028 Republic of Moldova*
[2]*Nordic* Institute *for Theoretical Physics (NORDITA) KTH and Stockholm University, Roslagstullsbacken 23, Stockholm, SE–106 91 Sweden*
E-mail: *sveatoslav.moscalenco@ifa.md, mliber@nordita.org*

https://doi.org/10.53081/mjps.2021.20-1.01 *CZU:538.9+539.17+539.14*

(Received October 14, 2020)

Abstract

In the present work, the Chern–Simons (CS) gauge field theory developed by Jackiw and Pi [8] and widely used to interpret the fractional quantum Hall effects, is applied to describe a two-dimensional (2D) electron–hole (e–h) system in a strong perpendicular magnetic field and under the influence of quantum point vortices creating the CS gauge field. Composite particles formed by electrons and holes with equal integer positive numbers ϕ of attached quantum point vortices are described by dressed field operators, which obey the Fermi or Bose statistics depending on even or odd numbers ϕ. It is shown that the phase operators, as well as the vector and scalar potentials of the CS gauge field, depend on the difference between the electron and hole density operators. They vanish in the mean field approximation, when the average values of electron and hole densities coincide. Nevertheless, even in this case, the quantum fluctuations of the CS gauge field lead to new physics of the 2D e–h system.

Keywords: Chern–Simons gauge field, quantum point vortices, electron–hole system, two-dimensional (2D), strong magnetic field.

Rezumat

În lucrarea de față, teoria câmpului de etalonare de tip Chern–Simons (CS), dezvoltată de Jackiw și Pi [8] și pe larg utilizată pentru a explica efectele cuantice fracționale de tip Hall, a fost aplicată pentru a descrie sistemul bidimensional (2D) compus din electroni și goluri (e–h) supuse unui câmp magnetic perpendicular puternic și sub influența vârtejurilor punctiforme cuantice care creează câmpul de etalonare de tip CS. Particulele compozite formate din electroni și din goluri cu numere pozitive întregi egale ϕ de vârtejuri punctiforme cuantice atașate sunt descrise de operatorii de câmp modificați, care se supun statisticilor Fermi sau Bose în dependență de numerele pare sau impare ϕ ale vârtejurilor atașate. Operatorii, care descriu fază, precum și potențialele vectoriale și scalare ale câmpului de etalonare de tip CS depind de diferența dintre operatorii de densitate ale electronilor și golurilor. Ele se anihilează în aproximarea câmpului mediu atunci când valorile medii ale densităților electronilor și golurilor coincid. Totuși, chiar și

în acest caz, fluctuațiile cuantice ale câmpului de etalonare de tip CS aduc la noi fenomene fizice în sistemul 2D e–h.

Cuvinte cheie: câmp de etalonare Chern–Simons, vârtejuri punctiforme cuantice, sistem din electroni și goluri bidimensionali (2D), câmp magnetic puternic.

1. Introduction

The Chern–Simons (CS) theory [1] is a quantum gauge theory using which some problems can be viewed from a different point of view providing better understanding. The most prominent of these problems are the change in the statistics of charged particles coupled to the CS field and the occurrence of a transverse conductivity, which directly makes the theory useful for describing the Hall effect. There is a deep analogy between CS gauge theories and quantum mechanical Landau levels of charge particles (electrons) in a magnetic field which have led to the understanding of the fractional quantum Hall effect [2–4] and simple understanding of the origin of massive gauge excitations in CS theories. It is worth mentioning that CS theories have important applications in the quantum field theory; the CS gauge theory can arise as a string theory, CS gravity theory, etc. The statistical transmutation leads to the physics of "anyons" [4], which are particles with generalized statistics neither fermionic nor bosonic that occur as excitations upon the ground state wave function (Laughlin wave function) of a quantum Hall system.

The CS Lagrangian for the (2+1)–dimensional space–time is $L_{CS} = -1/2\,\kappa\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho$, where $\varepsilon^{\mu\nu\rho}$ is a U(1) gauge field and κ is a constant. The CS Lagrangian is invariant for the gauge transformation $a_\mu \to a_\rho + \partial_\mu\chi$, which is seen from $\varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho \to \varepsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho + \varepsilon^{\mu\nu\rho}\partial_\mu\chi\partial_\nu a_\rho$. The last term here can be written as total derivative $dS = \int d^3x\varepsilon^{\mu\nu\rho}\partial_\mu(\chi\partial_\nu a_\rho)$, which means that it vanishes if there is no boundary, or we can neglect the boundary effect. The equation of motion for the a_μ fields is $\kappa\varepsilon^{\mu\nu\rho}\partial_\nu a_\rho = 0$; for the field strength tensor $f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$, we have $f_{\mu\nu} = 0$. This is a trivial result, which is of no interest from the point of view of physics until the a_μ CS field is coupled to a J^μ source, which is the conserved current of another real physical field.

Using a simple example, we recall the gauge theory notation for a planar system of electrons. The usual Lagrangian of the Maxwell gauge theory is

$$L_M = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_\mu J^\mu, \tag{1}$$

where $A_\mu = (A_0, \vec{A})$ is the gauge field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field antisymmetric tensor, J^μ is the conserved current, $\partial_\mu J^\mu = 0$. Lagrangian (1) is invariant under the $A_\mu \to A_\mu + \partial_\mu\Lambda$ gauge transformation and, accordingly, the Euler–Lagrange equations of motion $\partial_\mu F^{\mu\nu} = J^\nu$ are gauge invariant. The new situation is for 2+1 dimensions. In this case, the CS theory is a completely different type of gauge theory. It satisfies our usual criteria for a sensible gauge theory: it is Lorentz invariant, gauge invariant, and local. The CS Lagrangian is defined as follows:

$$\mathcal{L}_{CS} = \frac{\kappa}{2}\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - A_\mu J^\mu. \tag{2}$$

A gauge transformation changes the CS Lagrangian by a total space–time derivative:

$$\delta\mathcal{L}_{CS} = \frac{\kappa}{2}\partial_\mu(\lambda\varepsilon^{\mu\nu\rho}\partial_\nu A_\rho). \tag{3}$$

Therefore, if we can neglect boundary terms, then the respective CS action $S_{CS} = \int d^3x\mathcal{L}_{CS}$ is gauge invariant. Another important feature of CS Lagrangian (2) is that it is of the first order in space–time derivatives and, in 2+1 dimensions, the Lagrangian is quadratic in the gauge field. The Euler–Lagrange equations for CS Lagrangian are

$$\frac{\kappa}{2}\varepsilon^{\mu\nu\rho}F_{\nu\rho} = J^\mu, \text{ or } F_{\mu\nu} = \frac{1}{\kappa}\varepsilon_{\mu\nu\rho}J^\rho. \tag{4}$$

If we introduce a matter current $J^\mu = (\rho, \vec{J})$ and consider CS equations (4) coupled to matter fields, then the components of equation (4) are

$$\rho = \kappa B,\ J^i = \kappa\varepsilon^{ij}E_j. \tag{5}$$

The first equation (5) suggests that the charge density is locally proportional to the magnetic field, which means that the effect of a CS field is to relate magnetic flux to electric charge. Using the time derivative of the first equation in (5) $\dot{\rho} = \kappa\dot{B} = \kappa\varepsilon^{ij}\partial_i\dot{A}_j$ and the current conservation equation $\dot{\rho} + \partial_i J^i = 0$, we obtain

$$J^i = \kappa\varepsilon^{ij}\dot{A}_j + \varepsilon^{ij}\partial_j\chi, \tag{6}$$

which is the second equation in (5), transverse piece χ that can be identified with κA_0.

In this way, the effect of the CS coupling can be considered as a magnetic flux attached to the charge density in such a way that it everywhere follows the matter charge density. A feature of the CS theory is a magnetic flux attached to the charged particle fields together with the statistics transmutation. This feature is responsible for the appearance of CS fields in the composite boson or composite fermion models for the fractional quantum Hall effect, which involve quasiparticles that have magnetic fluxes attached to charged particles [5, 6].

Consider coupling of the Maxwell and CS Lagrangians, both of them producing gauge theories in 2+1 dimensions:

$$\mathcal{L}_{MCS} = -\frac{1}{4e^2}F_{\mu\nu}F^{\mu\nu} + \frac{\kappa}{2}\varepsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho. \tag{7}$$

The respective field equations

$$\partial_\mu F^{\mu\nu} + \frac{\kappa e^2}{2}\varepsilon^{\nu\alpha\beta}F_{\alpha\beta} = 0, \tag{8}$$

describe the propagation of a single degree of freedom with mass

$$m_{MCS} = \kappa e^2. \tag{9}$$

Here, κ is dimensionless and e^2 has mass dimension. If we write the equation of motion (8) in terms of the pseudovector "dual" field $\tilde{F}^{\mu} \equiv \varepsilon^{\mu\nu\rho}F_{\nu\rho}$, then we obtain

$$\left[\partial_{\mu}\partial^{\mu} + (\kappa e^2)^2\right]\tilde{F}^{\nu} = 0. \tag{10}$$

The origin of mass κe^2 is clearly seen from equation (10).

To better understand the importance of the CS theory, we consider the quantum mechanical analogy following [7]. In the gauge, $A_0 = 0$, the spatial components of gauge field $\vec{A}$ are conjugated to electric field $\vec{E}$, which satisfied equation $\nabla \cdot \vec{E} = \rho$, for which the "nondynamical" field A_0 is a Lagrange multiplier. We consider the structure of the Maxwell–CS theory Lagrangian:

$$L_{MCS} = \frac{1}{2e^2}E_i^2 - \frac{1}{2e^2}B^2 + \frac{\kappa}{2}\varepsilon^{ij}A_iA_j + kA_0B. \tag{11}$$

The "nondynamical" field A_0 is a Lagrange multiplier that can be regarded as a Lagrange multiplier in the Gauss law constraint:

$$\partial_i F^{i0} + \kappa e^2 \varepsilon^{ij}\partial_i A_j = 0. \tag{12}$$

This is the $\nu = 0$ component of the Euler–Lagrange equations (8). In the $A_0 = 0$ gauge, we identify A_i as "coordinate" fields with respective 'momentum' fields:

$$\Pi^i \equiv \frac{\partial L}{\partial \dot{A}_i} = \frac{1}{e^2}\dot{A}_i + \frac{\kappa}{2}\varepsilon^{ij}A_j. \tag{13}$$

Using the Legendre transformation, we write down the Hamiltonian

$$\begin{gathered}\mathcal{H}_{MCS} = \Pi^i\dot{A}_i - L = \\ \left(\frac{e^2}{2}\Pi^i - \frac{\kappa}{2}\varepsilon^{ij}A_j\right)^2 + \frac{1}{2e^2}B^2 + A_0(\partial_i\Pi^i + \kappa B).\end{gathered} \tag{14}$$

If we consider the long wavelength limit of the Maxwell–CS Lagrangian, in which we can drop all spatial derivatives, then the resulting Lagrangian will be as follows:

$$L = \frac{1}{2e^2}\dot{A}_i^2 + \frac{\kappa}{2}\varepsilon^{ij}\dot{A}_iA_j. \tag{15}$$

This is exactly the Lagrangian for a nonrelativistic charged particle moving in the plane in the presence of a uniform external magnetic field B perpendicular to the plane:

$$L = \frac{1}{2}\dot{x}_i^2 + \frac{b}{2}\varepsilon^{ij}\dot{x}_ix_j. \tag{16}$$

For momentum, we have

$$p_i = \frac{\partial L}{\partial \dot{x}_i} = m\dot{x}_i + \frac{B}{2}\varepsilon^{ij}x_j, \tag{17}$$

and the Hamiltonian is

$$\mathrm{H} = \mathrm{p_i \dot{x}_i - L} = \frac{1}{2\mathrm{m}}(\mathrm{p_i} - \frac{\mathrm{b}}{2}\varepsilon^{\mathrm{ij}}\mathrm{x_j})^2 = \frac{\mathrm{m}}{2}\mathrm{u_i^2}. \tag{18}$$

The quantum commutation relations $[\mathrm{x_i}, \mathrm{p_j}] = \mathrm{i}\delta_{\mathrm{ij}}$ imply that the velocities do not commute, i.e. $[\mathrm{v_i}, \mathrm{v_j}] = -\mathrm{i}\varepsilon_{\mathrm{ij}}\mathrm{b}/\mathrm{m}^2$. This shows the quantum mechanical analogy to the classic Landau problem of electrons moving in the plane in the presence of an external uniform magnetic field perpendicular to the plane. In the latter case, fields $\mathrm{A_i}(\vec{\mathrm{x}}, \mathrm{t})$ and $\Pi^{\mathrm{i}}(\vec{\mathrm{x}}, \mathrm{t})$ in Hamiltonian (14) satisfy classical equal-time Poisson brackets: $\left[\mathrm{A_i(x)}, \Pi^{\mathrm{j}}(\mathrm{y})\right] = \mathrm{i}\delta_{\mathrm{i}}^{\mathrm{j}}\delta(\vec{\mathrm{x}} - \vec{\mathrm{y}})$ and $\left[\mathrm{E_i(x)}, \mathrm{E_j(y)}\right] = \mathrm{i}\kappa \mathrm{e}^4\delta(\vec{\mathrm{x}} - \vec{\mathrm{y}})$.

There is a useful quantum mechanical analogy to the Landau problem of electrons moving in the plane with an external uniform magnetic field perpendicular to the plane; it is of special interest in quantum Hall systems.

It is now necessary to introduce density operators for the CS gauge field in a two-component electron–hole (e–h) system. We use the classical and quantum nonrelativistic CS theory for a two-dimensional N–body system of point particles, which was developed by Jackiw and Pi [8], to describe a 2D e–h system in a strong perpendicular magnetic field under the influence of quantum point vortices creating the CS gauge field. Phase operator $\hat{\omega}(\vec{r})$ of the CS field was introduced as a coherent summation of angles $\theta(\vec{r}-\vec{r}^{\,/})$ formed with the in-plane *x*-axis by reference vectors $(\vec{r}-\vec{r}^{\,/})$, which determine positions $\vec{r}^{\,/}$ of the particles creating the gauge field at point $\vec{r}$. It was pointed in [8] that angles $\theta(\vec{r}-\vec{r}^{\,/})$ are ill determined because arctangent is a multivalued function. However, this deficiency was compensated by the fact that the summation of the angles was weighted in [8] by density operators $\hat{\rho}(\vec{r}^{\,/})$ of the charged particles as follows:

$$\hat{\omega}(\vec{r}) = -\frac{\phi e}{\alpha}\int d\vec{r}^{\,/}\theta(\vec{r}-\vec{r}^{\,/})\hat{\rho}(\vec{r}^{\,/}); \;\; \theta(\vec{r}-\vec{r}^{\,/}) = \arctan\left(\frac{y-y^{/}}{x-x^{/}}\right). \tag{19}$$

Here, ϕ is an integer, positive number and α is the fine structure constant $\alpha = \mathrm{e}^2/\hbar\mathrm{c} = 1/137$. The integer value of ϕ is another factor, which makes the dressed field operators to be well defined. Since we are interested in the generalization of the CS theory from a one-component electron gas to a two–component e–h system, we repeat the main statements of the CS theory in a new version by introducing a supplementary label $i = e, h$ denoting electrons and holes from the very beginning. Partial field operators $\Psi_i(r)$ and $\Psi_i^+(r)$ lead to partial phase operators $\hat{\omega}_i(\vec{r})$ and to partial vector potential operators $\hat{\vec{a}}_i(\vec{r})$ taking into account the electrical charges of the electrons (-*e*) and the holes (+*e*).

The bare field operators will be denoted as $\hat{\Psi}_i^0(\vec{r})$ and $\Psi_i^{0+}(\vec{r})$ with supplementary label zero, whereas the dressed field operator $\hat{\Psi}_i(\vec{r})$ and $\Psi_i^+(\vec{r})$ will be written without it. Note that, while the bare and dressed field operators are different, their density operators $\hat{\rho}_i(\vec{r})$ and $\hat{\rho}_i^0(\vec{r})$

coincide:

$$\hat{\rho}_i(\vec{r}) = \hat{\Psi}_i^+(\vec{r})\hat{\Psi}_i(\vec{r}) = \hat{\Psi}_i^{+0}(\vec{r})\hat{\Psi}_i^0(\vec{r}) = \hat{\rho}_i^0(\vec{r}) \tag{20}$$

Below, we will show that this property is a consequence of the unitary transformation $\hat{u}^+(r)\hat{u}(r)=1$ and it concerns any operators, which are analytical functions of the density operators as follows: $f(\hat{\rho}_i(\vec{r})) = f(\hat{\rho}_i^0(\vec{r}))$.

For example, partial phase operators $\hat{\omega}_i(\vec{r})$ and partial vector potential operators $\hat{\vec{a}}_i(\vec{r})$ show this property:

$$\begin{gathered}
\hat{\omega}_i(\vec{r}) = -\frac{\phi e}{\alpha}\int d^2\vec{r}^{/}\theta(\vec{r}-\vec{r}^{/})\hat{\rho}_i(\vec{r}^{/}) = \hat{\omega}_i^0(\vec{r}),\\
\hat{\vec{a}}_i(\vec{r}) = \vec{\nabla}_{\vec{r}}\hat{\omega}_i(\vec{r}) = -\frac{\phi e}{\alpha}\int d^2\vec{r}^{/}\vec{\nabla}_{\vec{r}}\theta(\vec{r}-\vec{r}^{/})\hat{\rho}_i(\vec{r}^{/}) = \hat{\vec{a}}_i^0(\vec{r}),\\
\hat{\omega}(\vec{r}) = \hat{\omega}_e(\vec{r}) - \hat{\omega}_h(\vec{r}),\ \hat{\vec{a}}(\vec{r}) = \hat{\vec{a}}_e(\vec{r}) - \hat{\vec{a}}_h(\vec{r}),
\end{gathered} \tag{21}$$

and, as was pointed in [8], the $\theta(\vec{r}-\vec{r}^{/})$ function is ill determined. The differences added in (21) give rise to the resultant phase [9, 10] and vector potential operators [11] created by the integer e–h system [12].

Phase operators $\hat{\omega}_i(\vec{r})$ and $\hat{\omega}(\vec{r})$ are singular values because they are expressed in terms of a multivalued function, such as arctangent, and therefore as follows:

$$\begin{gathered}
\vec{\nabla}_{\vec{r}}\theta(\vec{r}-\vec{r}^{/}) = -\vec{\nabla}_{\vec{r}}\times\ln|\vec{r}-\vec{r}^{/}|;\ \vec{\nabla}\times\theta(\vec{r}-\vec{r}^{/}) = \vec{\nabla}\ln(\vec{r}-\vec{r}^{/}),\\
\Delta_{\vec{r}}\theta(\vec{r}-\vec{r}^{/}) = 0,\ \Delta_{\vec{r}}\ln|\vec{r}-\vec{r}^{/}| = 2\pi\delta^2(\vec{r}-\vec{r}^{/});
\end{gathered}$$

where

$$\begin{gathered}
\vec{\nabla}\times = \vec{e}_x\frac{\partial}{\partial y} - \vec{e}_y\frac{\partial}{\partial x};\ \vec{\nabla} = \vec{e}_x\frac{\partial}{\partial x} + \vec{e}_y\frac{\partial}{\partial y} = \frac{\vec{\in}_\theta}{r}\frac{\partial}{\partial\theta} + \in_{\vec{r}}\frac{\partial}{\partial r};\ \in_\theta = \frac{-\vec{e}_x y + \vec{e}_y x}{r};\ \in_{\vec{r}} = \frac{\vec{r}}{r},\\
\vec{\nabla}\times\vec{V} = \in^{ij}\partial_i V_j = \frac{\partial}{\partial x}V_y - \frac{\partial}{\partial y}V_x = S,\\
(\vec{\nabla}\times S)^i = \in^{ij}\partial_j S;\ \in^{12} = -\in^{21} = 1,\ \in^{n} = \in^{22} = 0.\\
\vec{\nabla}\hat{\vec{a}}(\vec{r}) = 0,\\
\vec{\nabla}\times\hat{\vec{a}}(\vec{r}) = \hat{\vec{b}}(\vec{r}) = \vec{\nabla}_{\vec{r}}\times\frac{\phi e}{\alpha}\int d^2\vec{r}^{/}\vec{\nabla}_{\vec{r}}\times\ln|\vec{r}-\vec{r}^{/}|\hat{\rho}(\vec{r}^{/}) =\\
= \frac{\phi e}{\alpha}\int d^2\vec{r}^{/}\Delta_{\vec{r}}\ln|\vec{r}-\vec{r}^{/}|\hat{\rho}(\vec{r}^{/}) = 2\pi\frac{\phi e}{\alpha}\hat{\rho}(\vec{r}),\\
\hat{\vec{b}}(\vec{r}) = 2\pi\frac{\phi e}{\alpha}\hat{\rho}(\vec{r}) = 2\pi\frac{\phi e}{\alpha}(\hat{\rho}_e(\vec{r}) - \hat{\rho}_h(\vec{r})).
\end{gathered} \tag{22}$$

It was pointed out in [8] that, in the 2D space, the curl of the vector is a scalar, whereas the curl of the scalar is a vector. These properties are shows by formulas (22), in particular, the Green function of the Laplacian in the 2D space is $(\ln\vec{r})/2\pi$. Other important data provided by formulas (22) is the effective magnetic field $\hat{\vec{b}}(\vec{r})$ expressed by $\vec{\nabla}\times\hat{\vec{a}}(\vec{r})$. It was shown in [8] that this magnetic field is created by quantum point vortices. In the case of a one-component electron gas, this supplementary magnetic field can compensate the external magnetic field. It will be shown below that, in the case of a two-component e–h system, this effective magnetic field has a special interesting property. It seems to vanish in the mean-field approximation when the average values of the density operators coincide $\langle\hat{\rho}_e(\vec{r})\rangle=\langle\hat{\rho}_h(\vec{r})\rangle$; however, its quantum fluctuations lead to unexpected physics of the 2D e–h system in a strong external magnetic field. Jackiw and Pi [8] gave a special attention to calculations involving the ill determined angle function $\theta(\vec{r}-\vec{r}^{\,/})$. They emphasized that, in the nonrelativistic quantum mechanics, the particles are points and density operator $\hat{\rho}(\vec{r})$ is localized at these points being a superposition of the δ functions. This fact plays a critical role in calculations involving the CS gauge field. For example, it permits interchanging the integration and the differentiation in the definition of the CS vector potential $\hat{\vec{a}}(\vec{r})$. Otherwise, it would be impossible to move the gradient with respect to $\vec{r}$ out of the integral on variable $\vec{r}^{\,/}$. In the general case, operators (21) are singular, because the $\theta(\vec{r}-\vec{r}^{\,/})$ angle is a multivalued function and the integration over the 2D $\vec{r}^{\,/}$ plane requires specifying the cut in $\vec{r}^{\,/}$ beginning in $\vec{r}$. However, the presence of density operator $\hat{\rho}(\vec{r}^{\,/})$ in the integral with δ-function eigenvalues leads to an exceptional situation, when $\vec{r}$-gradient can be moved with impunity outside the integral. Since the derivative of the $\theta(\vec{r}-\vec{r}^{\,/})$ function at point $\vec{r}=\vec{r}^{\,/}$ is ill defined, the authors of [8] proposed its regularization: $\vec{\nabla}_{\vec{r}}=\theta(\vec{r}-\vec{r}^{\,/})\Big|_{\vec{r}=\vec{r}^{\,/}}=0$.

To confirm the affirmation concerning the eigenvalues of the density operators, it is necessary to use the commutation relations between the field operators and the density operator:

$$\begin{aligned}&\left[\Psi_i(r),\hat{\rho}_i(\vec{r}^{\,/})\right]=\delta^2(\vec{r}-\vec{r}^{\,/})\hat{\Psi}_i(\vec{r}^{\,/});\ \left[\Psi_i^+(r),\hat{\rho}_i(\vec{r}^{\,/})\right]=-\delta^2(\vec{r}-\vec{r}^{\,/})\hat{\Psi}_i^+(\vec{r}^{\,/}),\\&\left[\hat{\rho}_i(\vec{r}),\hat{\rho}_j(\vec{r}^{\,/})\right]=0,\ \left[\hat{\rho}_i(\vec{r}),\hat{\omega}(\vec{r}^{\,/})\right]=0,\ \left[\hat{\omega}(\vec{r}^{\,/}),\hat{\vec{a}}(\vec{r})\right]=0.\end{aligned}\tag{23}$$

The proper functions of density operators $\hat{\rho}_i(\vec{r})$ can be introduced as follows:

$$\left|\hat{\Psi}_i(\vec{r}^{\,/})\right\rangle=\hat{\Psi}_i^+(\vec{r}^{\,/})|0\rangle,\ \left\langle\hat{\Psi}_i(\vec{r}^{\,/})\right|=\langle 0|\hat{\Psi}_i(\vec{r}^{\,/}),\tag{24}$$

where $|0\rangle$ is the ground state of the system. The action of the density operator on the $\left|\hat{\Psi}_i(\vec{r}^{\,/})\right\rangle$ function gives rise to the result

$$\begin{aligned}&\hat{\rho}_i(\vec{r})\left|\hat{\Psi}_i(\vec{r}^{\,/})\right\rangle=\hat{\Psi}_i^+(\vec{r})\hat{\Psi}_i(\vec{r})\hat{\Psi}_i^+(\vec{r}^{\,/})|0\rangle=\\&=\delta^2(\vec{r}-\vec{r}^{\,/})\hat{\Psi}_i^+(\vec{r}\ \ )|0\rangle=\delta^2(\vec{r}-\vec{r}^{\,/})\left|\hat{\Psi}_i(\vec{r}^{\,/})\right\rangle.\end{aligned}\tag{25}$$

It confirms that the eigenvalues of density operators $\hat{\rho}_i(\vec{r})$ have the form of δ-functions and that these operators play a decisive role in combating the deficiencies associated with the presence of the $\theta(\vec{r}-\vec{r}^{\,/})$ angle function. The integer, positive values of numbers ϕ in the definitions of operators (21) contribute also to the removal of the incertitude associated with the multivaluedness of the $\theta\left(\vec{r}-\vec{r}^{\,/}\right)$ angles.

The paper is organized as follows. Section 2 describes the unitary transformation operators that introduce the CS field into a two-component e–h system. In Section 3, we derive the Hamiltonian and the equations of motion for dressed field operators. The new properties of 2D magnetoexcitons under the influence of the CS gauge field are revealed in Section 4. Conclusions are given in Section 5.

2. Unitary Transformation Introducing the CS Gauge Field in a Two-Component Electron–Hole System

In a two-component e–h system, the electrons and the holes equally contribute to the creation of a unique and common CS gauge field, each of them acting with a proper electric charge. The resultant phase operator $\hat{\omega}\left(\vec{r}\right)$ and its gradient are algebraic sums of the partial electron and hole contributions

$$\begin{aligned}
&\hat{\omega}\left(\vec{r}\right)=\hat{\omega}_e\left(\vec{r}\right)-\hat{\omega}_h\left(\vec{r}\right)=-\frac{\phi e}{\alpha}\int d^2\vec{r}^{\,/}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\hat{\rho}\left(\vec{r}^{\,/}\right)=\hat{\omega}^{+}\left(\vec{r}\right),\\
&\hat{\vec{a}}\left(\vec{r}\right)=\hat{\vec{a}}_e\left(\vec{r}\right)-\hat{\vec{a}}_h\left(\vec{r}\right)=-\frac{\phi e}{\alpha}\int d^2\vec{r}^{\,/}\vec{\nabla}_{\vec{r}}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\hat{\rho}\left(\vec{r}^{\,/}\right)=\hat{\vec{a}}^{+}\left(\vec{r}\right),\\
&\hat{\rho}\left(\vec{r}\right)=\hat{\rho}_e\left(\vec{r}\right)-\hat{\rho}_h\left(\vec{r}\right)=\hat{\rho}^{+}\left(\vec{r}\right);\\
&\hat{\rho}_i\left(\vec{r}\right)=\hat{\Psi}_i^{+}\left(\vec{r}\right)\hat{\Psi}_i\left(\vec{r}\right)=\hat{\rho}_i^{+}\left(\vec{r}\right).
\end{aligned}\tag{26}$$

These expressions are true in the bare and dressed representations, no matter to which statistics—Fermi or Bose—obey the field operators. Unlike a one-component two-dimensional electron gas (2DEG), in this case, there are two subsystems with different electric charges. As a consequence, the resultant phase and vector potential operators compensate each other, so as to obtain a zero-gauge field in the mean-field approximation. It opens up the possibility of neglecting the effects arising due to the influence of the CS gauge field in the zero-order approximation and taking them into account in the next orders of the perturbation theory. In the case of a two-component e–h system, the unitary transformation introducing the CS gauge field looks as follows:

$$\hat{u}\left(r\right)=e^{\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\right)};\;\;\hat{u}^{+}\left(r\right)=e^{-\frac{ie}{\hbar c}\hat{\omega}\left(\vec{r}\right)};\;\hat{u}^{+}\left(r\right)\hat{u}\left(r\right)=1.\tag{27}$$

The bare electron and hole field operators will be denoted as $\hat{\Psi}_i^{0}\left(\vec{r}\right)$ and $\hat{\Psi}_i^{0+}\left(\vec{r}\right)$ with the supplementary zero label. They obey the Fermi statistics with the Fermi commutation relations

$$\begin{aligned}
&\hat{\Psi}_i^{0}\left(\vec{r}\right)\hat{\Psi}_j^{0+}\left(\vec{r}\right)+\hat{\Psi}_j^{0+}\left(\vec{r}\right)\hat{\Psi}_i^{0}\left(\vec{r}\right)=\delta_{ij}\delta^2\left(\vec{r}-\vec{r}^{\,/}\right),\\
&\hat{\Psi}_i^{0}\left(\vec{r}\right)\hat{\Psi}_j^{0}\left(\vec{r}^{\,/}\right)+\hat{\Psi}_j^{0}\left(\vec{r}^{\,/}\right)\hat{\Psi}_i^{0}\left(\vec{r}\right)=0.
\end{aligned}\tag{28}$$

The dressed electron and hole field operators creating the CS gauge field are written without the zero label and they are introduced in the form

$$\hat{\Psi}_e(\vec{r}) = u^+(\vec{r})\hat{\Psi}_e^0(\vec{r}),\ \hat{\Psi}_e^+(\vec{r}) = \hat{\Psi}_e^{0+}(\vec{r})u(\vec{r}),$$
$$\hat{\Psi}_h(\vec{r}) = \hat{u}(\vec{r})\hat{\Psi}_h^0(\vec{r}),\ \hat{\Psi}_h^+(\vec{r}) = \hat{\Psi}_h^{0+}(\vec{r})u^+(\vec{r}),$$
$$\hat{\rho}_i(\vec{r}) = \hat{\Psi}_i^+(\vec{r})\hat{\Psi}_i(\vec{r}) = \hat{\rho}_i^0(\vec{r}) = \hat{\Psi}_i^{0+}(\vec{r})\hat{\Psi}_i^0(\vec{r}), i = e, h;$$
$$\hat{\rho}_i^+(\vec{r}) = \hat{\rho}_i(\vec{r}),\ \hat{\rho}_i^{0+}(\vec{r}) = \hat{\rho}_i^0(\vec{r}),\ \left[\hat{\rho}_i(\vec{r}), \hat{\rho}_j(\vec{r}^{\,/})\right] = 0, \tag{29}$$
$$\hat{\rho}(\vec{r}) = \hat{\rho}_e(\vec{r}) - \hat{\rho}_h(\vec{r}) = \hat{\rho}^0(\vec{r}) = \hat{\rho}_e^0(\vec{r}) - \hat{\rho}_h^0(\vec{r}).$$

Due to the equality $\hat{\rho}(\vec{r}) = \hat{\rho}^0(\vec{r})$, the phase and vector potential operators determined by formulas (26) are the same in the bare and in the dressed representations:

$$\hat{\omega}(\vec{r}) = \hat{\omega}^0(\vec{r}),\ \hat{\omega}_i(\vec{r}) = \hat{\omega}_i^0(\vec{r}),$$
$$\hat{\vec{a}}(\vec{r}) = \hat{\vec{a}}^0(\vec{r}),\ \hat{\vec{a}}_i(\vec{r}) = \hat{\vec{a}}_i^0(\vec{r}),\ \hat{\rho}(\vec{r}) = \hat{\rho}^0(\vec{r}). \tag{30}$$

This also concerns other operators expressed as analytic functions depending on density operator $\hat{\rho}(\vec{r})$. Despite the fact that bare electron and hole field operators $\hat{\Psi}_i^0(\vec{r})$ and $\hat{\Psi}_i^{0+}(\vec{r})$ obey Fermi commutation relations (28), dressed field operators $\hat{\Psi}_i(\vec{r})$ and $\hat{\Psi}_i^+(\vec{r})$ satisfy the Fermi or Bose statistics depending on the even or odd integer, positive numbers ϕ introduced into the definitions of operators (26). To prove this statement, it is necessary to derive first the commutation relations between field operators $\hat{\Psi}_i(\vec{r})$ and $\hat{\Psi}_i^+(\vec{r})$ with density operators $\hat{\rho}_i(\vec{r})$ and the commutation relations between the field operators and unitary transformations operators $u(\vec{r})$ and $u^+(\vec{r})$. The first of them are as follows:

$$\left[\hat{\Psi}_i(\vec{r}), \hat{\rho}_i(\vec{r}^{\,/})\right] = \delta^2(\vec{r} - \vec{r}^{\,/})\hat{\Psi}_i(\vec{r}^{\,/});\ \left[\hat{\Psi}_i^+(\vec{r}^{\,/}), \hat{\rho}_i(\vec{r}^{\,/})\right] = -\delta^2(\vec{r} - \vec{r}^{\,/})\hat{\Psi}_i^+(\vec{r}^{\,/}). \tag{31}$$

They are the same as those in the case of the Fermi or Bose statistics. Thereupon, the following commutation relations can be obtained:

$$\hat{\Psi}_e(\vec{r})\hat{\omega}^n(\vec{r}^{\,/}) = \left[\hat{\omega}(\vec{r}^{\,/}) - \frac{\phi e}{\alpha}\theta(\vec{r}^{\,/} - \vec{r})\right]^n \hat{\Psi}_e(\vec{r}),$$
$$\hat{\Psi}_h(\vec{r})\hat{\omega}^n(\vec{r}^{\,/}) = \left[\hat{\omega}(\vec{r}^{\,/}) + \frac{\phi e}{\alpha}\theta(\vec{r}^{\,/} - \vec{r})\right]^n \hat{\Psi}_h(\vec{r}),$$
$$\hat{\Psi}_e^+(\vec{r})\hat{\omega}^n(\vec{r}^{\,/}) = \left[\hat{\omega}(\vec{r}^{\,/}) + \frac{\phi e}{\alpha}\theta(\vec{r}^{\,/} - \vec{r})\right]^n \hat{\Psi}_e^+(\vec{r}), \tag{32}$$
$$\hat{\Psi}_h^+(\vec{r})\hat{\omega}^n(\vec{r}^{\,/}) = \left[\hat{\omega}(\vec{r}^{\,/}) - \frac{\phi e}{\alpha}\theta(\vec{r}^{\,/} - \vec{r})\right]^n \hat{\Psi}_h^+(\vec{r}).$$

In this case, the commutation relations of field operators $\hat{\Psi}_i(\vec{r})$ with unitary transformation operators $\exp(\pm ie\hat{\omega}(\vec{r}^{\,/})/(\hbar c))$ will be

$$\hat{\Psi}_e(\vec{r})e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}=\sum_{n=0}^{\infty}\left(\frac{\pm ie}{\hbar c}\right)^n\frac{1}{n!}\hat{\Psi}_e(\vec{r})\hat{\omega}^n(\vec{r}^{/})=$$
$$=\sum_{n=0}^{\infty}\left(\frac{\pm ie}{\hbar c}\right)^n\frac{1}{n!}\left[\hat{\omega}(\vec{r}^{/})-\frac{\phi e}{\alpha}\theta(\vec{r}^{/}-\vec{r})\right]^n\hat{\Psi}_e(\vec{r})=$$
$$=e^{\mp i\phi\theta(\vec{r}^{/}-\vec{r})}e^{\frac{\pm ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}\hat{\Psi}_e(\vec{r}),$$
$$\hat{\Psi}_h(\vec{r})e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}=e^{\pm i\phi\theta(\vec{r}^{/}-\vec{r})}e^{\frac{\pm ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}\hat{\Psi}_h(\vec{r}),$$
$$\hat{\Psi}_e^{+}(\vec{r})e^{\frac{\pm ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}=e^{\pm i\phi\theta(\vec{r}^{/}-\vec{r})}e^{\frac{\pm ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}\hat{\Psi}_e^{+}(\vec{r}), \tag{33}$$
$$\hat{\Psi}_h^{+}(\vec{r})e^{\frac{\pm ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}=e^{\mp i\phi\theta(\vec{r}^{/}-\vec{r})}e^{\frac{\pm ie}{\hbar c}\hat{\omega}(\vec{r}^{/})}\hat{\Psi}_h^{+}(\vec{r}).$$

To prove the main statement concerning the statistics of the dressed field operators, we will start with the first equation (28) and transcribe it from the bare to dressed operators as follows:

$$\begin{aligned}
&\hat{\Psi}_e^{0}(\vec{r})\hat{\Psi}_e^{0+}(\vec{r}^{/})+\hat{\Psi}_e^{0+}(\vec{r}^{/})\hat{\Psi}_e^{0}(\vec{r})=\delta^2(\vec{r}-\vec{r}^{/})=\\
&=u\ (\vec{r})\hat{\Psi}_e(\vec{r})\hat{\Psi}_e^{+}(\vec{r}^{/})u^{+}(\vec{r}^{/})+\hat{\Psi}_e^{+}(\vec{r}^{/})u^{+}(\vec{r}^{/})u\ (\vec{r})\hat{\Psi}_e(\vec{r})=\\
&=\hat{\Psi}_e(\vec{r})\hat{\Psi}_e^{+}(\vec{r}^{/})u\ (\vec{r})u^{+}(\vec{r}^{/})e^{i\phi\theta(0)}e^{-i\phi\theta(\vec{r}-\vec{r}^{/})}+\\
&+\hat{\Psi}_e^{+}(\vec{r}^{/})\hat{\Psi}_e(\vec{r})u^{+}(\vec{r}^{/})u\ (\vec{r})e^{i\phi\theta(0)}e^{-i\phi\theta(\vec{r}^{/}-\vec{r})},\\
&u\ (\vec{r})u^{+}(\vec{r}^{/})=u^{+}(\vec{r}^{/})u\ (\vec{r}),\\
&\theta(\vec{r}^{/}-\vec{r})=\theta(\vec{r}-\vec{r}^{/})+\pi.
\end{aligned} \tag{34}$$

With account of the last two relations, equation (34) can be transcribed in the form

$$\begin{aligned}
&e^{i\phi\theta(0)}e^{-i\phi\theta(\vec{r}-\vec{r}^{/})}\left[\hat{\Psi}_e(\vec{r})\hat{\Psi}_e^{+}(\vec{r}^{/})+e^{-i\phi\pi}\hat{\Psi}_e^{+}(\vec{r}^{/})\hat{\Psi}_e(\vec{r})\right]\times\\
&\times u\ (\vec{r})u^{+}(\vec{r}^{/})=\delta^2(\vec{r}-\vec{r}^{/}).
\end{aligned} \tag{35}$$

It is equivalent to the commutation relation

$$\begin{aligned}
&\hat{\Psi}_e(\vec{r})\hat{\Psi}_e^{+}(\vec{r}^{/})+e^{-i\phi\pi}\hat{\Psi}_e^{+}(\vec{r}^{/})\hat{\Psi}_e(\vec{r})=\delta^2(\vec{r}-\vec{r}^{/}),\\
&e^{-i\phi\pi}=\cos\phi\pi-\mathrm{i}\sin(\phi\pi)=\begin{Bmatrix}1,\phi=0,2,4...,F\\-1,\phi=1,3,5..,\mathrm{B}\end{Bmatrix}.
\end{aligned} \tag{36}$$

The most important result of these calculation is the affirmation that CS gauge field operators $\hat{\Psi}_i^{+}(\vec{r})$ and $\hat{\Psi}_i(\vec{r})$ with $i=e,h$ obey the Fermi statistics in the case of the even integer, positive pair numbers ϕ and the Bose statistics in the case of odd integer positive numbers ϕ. It is an important result of the CS gauge field theory developed by Jackiw and Pi in [8].

3. Hamiltonian and Equations of Motion Describing the Dressed Operators of the CS Gauge Field

To obtain a Hamiltonian describing the interaction of the composite particles expressed in terms of dressed field operators $\hat{\Psi}_i^{+}(\vec{r})$ and $\hat{\Psi}_i(\vec{r})$ and deduce their equations of motion, we will start with the respective expressions for bare field operators $\hat{\Psi}_i^{0+}(\vec{r})$ and $\hat{\Psi}_i^{0}(\vec{r})$. The Hamiltonian describing bare 2D electrons and holes in an external perpendicular magnetic field and interacting by the Coulomb forces obtained in [13] looks as follows:

$$
\begin{aligned}
&\hat{H}^0 = \hat{K}^0 + \hat{H}^0_{Coul},\\
&\hat{K}^0 = \frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\hat{\Psi}_e^{0+}(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/} + \frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})\right)^2 \hat{\Psi}_e^{0+}(\vec{r}^{\,/}) +\\
&+\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\hat{\Psi}_h^{0+}(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/} - \frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})\right)^2 \hat{\Psi}_h^{0+}(\vec{r}^{\,/}),\\
&\hat{H}^0_{Coul} = \frac{1}{2}\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}V_{Coul}(\vec{r}^{\,/}-\vec{r}^{\,//})\hat{\Psi}_e^{0+}(\vec{r}^{\,/})\hat{\rho}_e^0(\vec{r}^{\,//})\hat{\Psi}_e^0(\vec{r}^{\,/}) +\\
&+\frac{1}{2}\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}V_{Coul}(\vec{r}^{\,/}-\vec{r}^{\,//})\hat{\Psi}_h^{0+}(\vec{r}^{\,/})\hat{\rho}_h^0(\vec{r}^{\,//})\hat{\Psi}_h^0(\vec{r}^{\,/}) -\\
&-\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}V_{Coul}(\vec{r}^{\,/}-\vec{r}^{\,//})\hat{\Psi}_e^{0+}(\vec{r}^{\,/})\hat{\rho}_h^0(\vec{r}^{\,//})\hat{\Psi}_e^0(\vec{r}^{\,/}).
\end{aligned}
\tag{37}
$$

Here, $\vec{A}(\vec{r}^{\,/})$ is the vector potential created by an external magnetic field perpendicular to the layer. In the Landau gauge description, it has the form of $\vec{A}(\vec{r}) = (-B\cdot y, 0, 0)$, where B is the magnetic field strength. The vector potential obeys the following condition: $\vec{\nabla}\vec{A}(\vec{r}) = 0$. The Coulomb interaction potential in a 2D system can be represented as

$$
V_{Coul}(\vec{r}) = \sum_{\vec{Q}} V_{\vec{Q}} e^{i\vec{Q}\vec{r}},\ V(\vec{Q}) = \frac{2\pi e^2}{\varepsilon_0 S|\vec{Q}|},\ W(\vec{Q}) = V(\vec{Q})e^{-\frac{Q^2 l_0^2}{2}};\ l_0^2 = \frac{\hbar c}{eB}. \tag{38}
$$

Here, S is the layer surface area, ε_0 is the effective dielectric constant, and l_0 is the magnetic length. Coefficient $W(\vec{Q})$, along with coefficient $V(\vec{Q})$, was introduced into (38); it will be used below.

Bare density operators $\hat{\rho}_i^0(\vec{r})$ were introduced by formula (20). Schrodinger equations for bare operators $\hat{\Psi}_e^0(\vec{r})$ and $\hat{\Psi}_h^0(\vec{r})$ were derived in [13], and we just recall them:

$$
\begin{aligned}
& i\hbar\frac{d\hat{\Psi}_e^0(\vec{r})}{dt}=\left[\hat{\Psi}_e^0(\vec{r}),\hat{H}^0\right]=\frac{\hbar^2}{2m_e}\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_e^0(\vec{r})+\\
& +\int d^2\vec{r}^{/}V_{Coul.}\left(\vec{r}-\vec{r}^{/}\right)\hat{\rho}^0\left(\vec{r}^{/}\right)\hat{\Psi}_e^0(\vec{r}),\\
& i\hbar\frac{d\hat{\Psi}_h^0(\vec{r})}{dt}=\left[\hat{\Psi}_h^0(\vec{r}),\hat{H}^0\right]=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_h^0(\vec{r})-\\
& -\int d^2\vec{r}^{/}V_{Coul.}\left(\vec{r}-\vec{r}^{/}\right)\hat{\rho}^0\left(\vec{r}^{/}\right)\hat{\Psi}_h^0(\vec{r}),\\
& i\hbar\frac{d\hat{\Psi}_e^{0+}(\vec{r})}{dt}=\left[\hat{\Psi}_e^{0+}(\vec{r}),\hat{H}^0\right]=-\frac{\hbar^2}{2m_e}\left(i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_e^{0+}(\vec{r})-\\
& -\int d^2\vec{r}^{/}V_{Coul.}\left(\vec{r}-\vec{r}^{/}\right)\hat{\Psi}_e^{0+}(\vec{r})\hat{\rho}^0\left(\vec{r}^{/}\right),\\
& i\hbar\frac{d\hat{\Psi}_h^{0+}(\vec{r})}{dt}=-\frac{\hbar^2}{2m_h}\left(i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_h^{0+}(\vec{r})+\int d^2\vec{r}^{/}V_{Coul.}\left(\vec{r}-\vec{r}^{/}\right)\hat{\Psi}_h^{0+}(\vec{r})\hat{\rho}^0\left(\vec{r}^{/}\right);\\
& \hat{\rho}^{0+}\left(\vec{r}^{/}\right)=\hat{\rho}^0\left(\vec{r}^{/}\right).
\end{aligned}
\tag{39}
$$

Time derivatives $d(\hat{\rho}_i^0(\vec{r}))/dt$ and the continuity equations were derived using equations of motion

$$
\begin{aligned}
& \frac{d}{dt}\hat{\rho}_i^0(\vec{r})=\frac{d}{dt}\left(\hat{\Psi}_i^{0+}(\vec{r})\hat{\Psi}_i^0(\vec{r})\right)=-\vec{\nabla}\hat{\vec{J}}_i^0(\vec{r});\ i=e,h.\\
& \hat{\vec{J}}_e^0(\vec{r})=\frac{\hbar}{2m_e i}\left(\hat{\Psi}_e^{0+}(\vec{r})\vec{\nabla}\hat{\Psi}_e^0(\vec{r})-\vec{\nabla}\hat{\Psi}_e^{0+}(\vec{r})\hat{\Psi}_e^0(\vec{r})\right)+\frac{e}{m_e c}\vec{A}(\vec{r})\hat{\rho}_e^0(\vec{r}),\\
& \hat{\vec{J}}_h^0(\vec{r})=\frac{\hbar}{2m_h i}\left(\hat{\Psi}_h^{0+}(\vec{r})\vec{\nabla}\hat{\Psi}_h^0(\vec{r})-\vec{\nabla}\hat{\Psi}_h^{0+}(\vec{r})\hat{\Psi}_h^0(\vec{r})\right)-\frac{e}{m_h c}\vec{A}(\vec{r})\hat{\rho}_h^0(\vec{r}).
\end{aligned}
\tag{40}
$$

The equations of motion for dressed field operators $\hat{\Psi}_i^+(\vec{r})$ and $\hat{\Psi}_i(\vec{r})$ can be obtained taking into account equation (39) and time derivatives of unitary transformation operators $\hat{u}(\vec{r})=\hat{u}^0(\vec{r})$ and $\hat{u}^+(\vec{r})=\hat{u}^{0+}(\vec{r})$, which coincide in the bare and dressed representations as well as operators $\hat{\rho}_i(\vec{r})=\hat{\rho}_i^0(\vec{r})$, $\hat{\omega}_i(\vec{r})=\hat{\omega}_i^0(\vec{r})$. Therefore, one of them can be used in calculations without supplementary specifications.

Following the procedure used in [13], we obtain two equations:

$$
\begin{aligned}
& \frac{i\hbar d\hat{\Psi}_e(\vec{r})}{dt}=i\hbar\frac{d}{dt}\left(\hat{u}^+(\vec{r})\hat{\Psi}_e^0(\vec{r})\right)=i\hbar\frac{d}{dt}\hat{u}^+(\vec{r})\cdot\hat{\Psi}_e^0(\vec{r})+\hat{u}^+(\vec{r})\frac{i\hbar d}{dt}\hat{\Psi}_e^0(\vec{r}),\\
& \frac{i\hbar d\hat{\Psi}_h(\vec{r})}{dt}=i\hbar\frac{d}{dt}\left(\hat{u}(\vec{r})\hat{\Psi}_h^0(\vec{r})\right)=i\hbar\frac{d}{dt}\hat{u}(\vec{r})\cdot\hat{\Psi}_h^0(\vec{r})+\hat{u}(\vec{r})\frac{i\hbar d}{dt}\hat{\Psi}_h^0(\vec{r}).
\end{aligned}
\tag{41}
$$

To obtain time derivatives of unitary transformation operators $\exp(\pm ie\hat{\omega}(\vec{r})/(\hbar c))$, we take into account that operators $d\hat{\omega}(\vec{r})/dt$ and $\hat{\omega}(\vec{r})$ do not commute. This was pointed out by Jackiw and Pi in [8] and demonstrated in [13] as follows:

$$\begin{aligned}
&\left[\frac{d\hat{\omega}(\vec{r})}{dt},\hat{\omega}(\vec{r})\right]=-i\hat{L}(\vec{r})=\left(\frac{\phi e}{\alpha}\right)^2\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\times\\
&\times\theta\left(\vec{r}-\vec{r}^{\,//}\right)\left[\frac{d\hat{\rho}\left(\vec{r}^{\,/}\right)}{dt},\hat{\rho}\left(\vec{r}^{\,//}\right)\right]=\left(\frac{\phi e}{\alpha}\right)^2\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\theta\left(\vec{r}-\vec{r}^{\,//}\right)\times\\
&\times\left\{\left[\frac{d\hat{\rho}_e\left(\vec{r}^{\,/}\right)}{dt},\hat{\rho}_e\left(\vec{r}^{\,//}\right)\right]+\left[\frac{d\hat{\rho}_h\left(\vec{r}^{\,/}\right)}{dt},\hat{\rho}_h\left(\vec{r}^{\,//}\right)\right]\right\}=\left(\frac{\phi e}{\alpha}\right)^2\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}\times\\
&\times\vec{\nabla}^{/}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\theta\left(\vec{r}-\vec{r}^{\,//}\right)\left\{\left[\hat{\vec{J}}_e\left(\vec{r}^{\,/}\right),\hat{\rho}_e\left(\vec{r}^{\,//}\right)\right]+\left[\hat{\vec{J}}_h\left(\vec{r}^{\,/}\right)\hat{\rho}_h\left(\vec{r}^{\,//}\right)\right]\right\},
\end{aligned}\tag{42}$$

where we used expressions (40), which lead to the form

$$\begin{aligned}
&\hat{L}(\vec{r})=\frac{\hbar}{2m_e}\hat{M}_e(\vec{r})+\frac{\hbar}{2m_h}\hat{M}_h(\vec{r}),\\
&\hat{M}_i(\vec{r})=\left(\frac{\phi e}{\alpha}\right)^2\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}\vec{\nabla}^{/}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\theta\left(\vec{r}-\vec{r}^{\,//}\right)\times\\
&\times\left\{\left[\hat{\Psi}_i^{0+}\left(\vec{r}^{\,/}\right)\vec{\nabla}^{/}\hat{\Psi}_i^{0}\left(\vec{r}^{\,/}\right),\hat{\rho}_i^0\left(\vec{r}^{\,//}\right)\right]-\left[\vec{\nabla}^{/}\hat{\Psi}_i^{0+}\left(\vec{r}^{\,/}\right)\hat{\Psi}_i^{0}\left(\vec{r}^{\,/}\right),\hat{\rho}_i^0\left(\vec{r}^{\,//}\right)\right]\right\};\ i=e,h.
\end{aligned}\tag{43}$$

The commutation relations that were substituted into formulas (43) were calculated in [13] and look as follows:

$$\begin{aligned}
&\left[\hat{\Psi}_i^{0+}\left(\vec{r}^{\,/}\right)\vec{\nabla}^{/}\hat{\Psi}_i^{0}\left(\vec{r}^{\,/}\right),\hat{\rho}_i^0\left(\vec{r}^{\,//}\right)\right]=\hat{\Psi}_i^{0+}\left(\vec{r}^{\,/}\right)\vec{\nabla}^{/}\left(\delta^2\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}_i^{0}\left(\vec{r}^{\,//}\right)\right)-\\
&-\delta^2\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}_i^{0+}\left(\vec{r}^{\,//}\right)\vec{\nabla}^{/}\hat{\Psi}_i^{0}\left(\vec{r}^{\,/}\right),\\
&\left[\vec{\nabla}^{/}\hat{\Psi}_i^{0+}\left(\vec{r}^{\,/}\right)\hat{\Psi}_i^{0}\left(\vec{r}^{\,/}\right),\hat{\rho}_i^0\left(\vec{r}^{\,//}\right)\right]=-\vec{\nabla}^{/}\left(\delta^2\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}_i^{0+}\left(\vec{r}^{\,//}\right)\right)\hat{\Psi}_i^{0}\left(\vec{r}^{\,/}\right)+\\
&+\vec{\nabla}^{/}\hat{\Psi}_i^{0+}\left(\vec{r}^{\,/}\right)\delta^2\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}_i^{0}\left(\vec{r}^{\,//}\right),\ i=e,h.
\end{aligned}\tag{44}$$

The $\Delta^{/}\theta\left(\vec{r}-\vec{r}^{\,/}\right)=0$ and $\left(\vec{\nabla}^{/}\theta\left(\vec{r}-\vec{r}^{\,/}\right)\right)^2=\left|\vec{r}-\vec{r}^{\,/}\right|^{-2}$ properties help to effectuate the next calculations, which lead to the expressions

$$\begin{aligned}
&\hat{M}_i(\vec{r})=2\left(\frac{\phi e}{\alpha}\right)^2\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_i^0\left(\vec{r}^{\,/}\right)}{\left|\vec{r}-\vec{r}^{\,/}\right|^2},\ i=e,h;\\
&\hat{L}(\vec{r})=\left(\frac{\phi e}{\alpha}\right)\hbar\left[\frac{1}{m_e}\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_e^0\left(\vec{r}^{\,/}\right)}{\left|\vec{r}-\vec{r}^{\,/}\right|^2}+\frac{1}{m_h}\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_h^0\left(\vec{r}^{\,/}\right)}{\left|\vec{r}-\vec{r}^{\,/}\right|^2}\right],\\
&\left[\hat{L}(\vec{r}),\hat{\omega}(\vec{r})\right]=\left[\hat{L}(\vec{r}),\hat{\rho}_i\ (\vec{r})\right]=0.
\end{aligned}\tag{45}$$

To derive time derivatives of the unitary transformation operators, we will use the following formula:

$$\frac{d}{dt}e^{\pm\frac{ie}{\hbar c}}\hat{\omega}(\vec{r})=\sum_{n=0}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{n}\frac{1}{n!}\frac{d}{dt}\hat{\omega}^{n}(\vec{r}). \tag{46}$$

The first steps are the following results:

$$\begin{aligned}
&\frac{d}{dt}\hat{\omega}^{2}(\vec{r})=2\hat{\omega}(\vec{r})\frac{d\hat{\omega}(\vec{r})}{dt}-i\hat{L}(\vec{r})=2\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\omega}(\vec{r})+i\hat{L}(\vec{r}),\\
&\frac{d}{dt}\hat{\omega}^{3}(\vec{r})=3\omega^{2}(\vec{r})\frac{d\omega(\vec{r})}{dt}-3i\hat{L}(\vec{r})\hat{\omega}(\vec{r})=3\frac{d\omega(\vec{r})}{dt}\hat{\omega}^{2}(\vec{r})+3i\hat{L}(\vec{r})\hat{\omega}(\vec{r}),\\
&\frac{d}{dt}\hat{\omega}^{n}(\vec{r})=n\omega^{(n-1)}(\vec{r})\frac{d\omega(\vec{r})}{dt}-iX_{n}\hat{L}(\vec{r})\hat{\omega}^{(n-2)}(\vec{r})=\\
&=n\frac{d\omega(\vec{r})}{dt}\hat{\omega}^{(n-1)}(\vec{r})+iX_{n}\hat{L}(\vec{r})\hat{\omega}^{(n-2)}(\vec{r}),\\
&X_{n}=(n-1)+X_{n-1}+\frac{n(n-1)}{2},\ n\geq 2.
\end{aligned} \tag{47}$$

The last equation was used as follows:

$$\begin{aligned}
&\frac{d}{dt}e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}=\left(\pm\frac{ie}{\hbar c}\right)\frac{d\hat{\omega}(\vec{r})}{dt}+\sum_{n=2}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{n}\frac{1}{n!}\frac{d}{dt}\hat{\omega}(\vec{r})^{n}=\\
&=\left(\pm\frac{ie}{\hbar c}\right)\frac{d\hat{\omega}(\vec{r})}{dt}+\sum_{k=1}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{(k+1)}\frac{1}{\mathrm{k}!}\hat{\omega}^{k}(\vec{r})\frac{d\hat{\omega}(\vec{r})}{dt}-\\
&-i\hat{L}(\vec{r})\sum_{m=0}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{(m+2)}\frac{X_{(m+2)}}{(m+2)!}\hat{\omega}^{m}(\vec{r})=\\
&=\left(\pm\frac{ie}{\hbar c}\right)\frac{d\hat{\omega}(\vec{r})}{dt}+\frac{d\hat{\omega}(\vec{r})}{dt}\sum_{k=1}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{(k+1)}\frac{1}{\mathrm{k}!}\hat{\omega}^{k}(\vec{r})+\\
&+i\hat{L}(\vec{r})\sum_{m=0}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{(m+2)}\frac{X_{(m+2)}}{(m+2)!}\hat{\omega}^{m}(\vec{r})=\\
&=e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}\left[\left(\pm\frac{ie}{\hbar c}\right)\frac{d\hat{\omega}(\vec{r})}{dt}+\frac{i}{2}\left(\frac{e}{\hbar c}\right)^{2}\hat{L}(\vec{r})\right]=\\
&=\left[\left(\pm\frac{ie}{\hbar c}\right)\frac{d\hat{\omega}(\vec{r})}{dt}-\frac{i}{2}\left(\frac{e}{\hbar c}\right)^{2}\hat{L}(\vec{r})\right]e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}\times\\
&\times\frac{X_{(m+2)}}{(m+2)!}=\frac{1}{2}\cdot\frac{1}{m!},\ m\geq 0.
\end{aligned} \tag{48}$$

We can now calculate the commutation relation between time derivative $d\hat{\omega}(\vec{r})/dt$ and unitary transformation operators $\exp(\pm ie\hat{\omega}(\vec{r})/(\hbar c))$:

$$\left[\frac{d\hat{\omega}(\vec{r})}{dt}, e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}\right] = \sum_{n=0}^{\infty}\left(\pm\frac{ie}{\hbar c}\right)^{n}\cdot\frac{1}{n!}\left[\frac{d\hat{\omega}(\vec{r})}{dt}, \hat{\omega}^{n}(\vec{r})\right]. \tag{49}$$

To clarify this issue, we recall the basic equalities

$$\left[\frac{d\hat{\omega}(\vec{r})}{dt}, \hat{\omega}(\vec{r})\right] = -i\hat{L}(\vec{r}),\ \left[\frac{d\hat{\omega}^{2}(\vec{r})}{dt}, \hat{\omega}(\vec{r})\right] = -2i\hat{L}(\vec{r})\hat{\omega}(\vec{r}).$$

They lead to the recurrent formula

$$\left[\frac{d\hat{\omega}(\vec{r})}{dt}, \hat{\omega}^{n}(\vec{r})\right] = -in\hat{L}(\vec{r})\hat{\omega}^{n-1}(\vec{r}), \tag{50}$$

which solves the problem:

$$\left[\frac{d\hat{\omega}(\vec{r})}{dt}, e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}\right] = \left(\pm\frac{ie}{\hbar c}\right)\hat{L}(\vec{r})e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}. \tag{51}$$

Next, to derive the equations of motion for dressed field operators $\hat{\Psi}_{i}(\vec{r})$ and $\hat{\Psi}_{i}^{+}(\vec{r})$, we consider equations

$$\begin{aligned} i\hbar\frac{d}{dt}\hat{\Psi}_{e}(\vec{r}) &= i\hbar\frac{d}{dt}\left(\hat{u}^{+}(\vec{r})\hat{\Psi}_{e}^{0}(\vec{r})\right) = i\hbar\frac{d\hat{u}^{+}(\vec{r})}{dt}\hat{\Psi}_{e}^{0}(\vec{r}) + \hat{u}^{+}(\vec{r})i\hbar\frac{d\hat{\Psi}_{e}^{0}(\vec{r})}{dt}, \\ i\hbar\frac{d}{dt}\hat{\Psi}_{h}(\vec{r}) &= i\hbar\frac{d}{dt}\left(\hat{u}(\vec{r})\hat{\Psi}_{h}^{0}(\vec{r})\right) = i\hbar\frac{d\hat{u}(\vec{r})}{dt}\hat{\Psi}_{h}^{0}(\vec{r}) + \hat{u}(\vec{r})i\hbar\frac{d\hat{\Psi}_{h}^{0}(\vec{r})}{dt}. \end{aligned} \tag{52}$$

We recall that operators $\hat{\omega}(\vec{r})$ and $d\hat{\omega}(\vec{r})/dt$ do not commute and their commutation equals to

$$\begin{aligned} &\left[\frac{d\hat{\omega}(\vec{r})}{dt}, \hat{\omega}(\vec{r})\right] = -i\hat{L}(\vec{r}), \\ &\hat{L}^{+}(\vec{r}) = \hat{L}(\vec{r}),\ \left[\hat{L}(\vec{r}), \hat{\rho}_{i}(\vec{r})\right] = 0,\ \left[\hat{L}(\vec{r}), \hat{\omega}(\vec{r}^{\prime})\right] = 0. \end{aligned} \tag{53}$$

Similar to as it was done in the derivation of equation (48), we can write

$$\begin{aligned} i\hbar\frac{d\hat{u}(\vec{r})}{dt} &= \hat{u}(\vec{r})\left[-\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt} - \frac{e^{2}}{2\hbar c^{2}}\hat{L}(\vec{r})\right] = \\ &= \left[-\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt} + \frac{e^{2}}{2\hbar c^{2}}\hat{L}(\vec{r})\right]\hat{u}(\vec{r}), \\ \hat{u}(\vec{r}) &= e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}, \end{aligned}$$

$$i\hbar\frac{d\hat{u}^{+}(\vec{r})}{dt}=\hat{u}^{+}(\vec{r})\left[\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}-\frac{e^2}{2\hbar c^2}\hat{L}(\vec{r})\right]=$$
$$=\left[\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}+\frac{e^2}{2\hbar c^2}\hat{L}(\vec{r})\right]\hat{u}^{+}(\vec{r}), \tag{54}$$
$$\hat{u}^{+}(\vec{r})=e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}.$$

Below, we will use formulas describing the action of operator $\left(-i\vec{\nabla}\right)$ on unitary transformation operators $\exp(\pm ie\hat{\omega}(\vec{r})/(\hbar c))$:

$$\left(-i\vec{\nabla}\right)e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}=\pm\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})e^{\pm\frac{ie}{\hbar c}\hat{\omega}(\vec{r})};\ \ \hat{\vec{a}}(\vec{r})=\vec{\nabla}\hat{\omega}(\vec{r}),$$
$$e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_e^0(\vec{r})=\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_e(\vec{r}),$$
$$e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_h^0(\vec{r})=\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_h(\vec{r}). \tag{55}$$
$$\hat{\Psi}_e(\vec{r})=u^{+}(\vec{r})\hat{\Psi}_e^0(\vec{r}),\hat{\Psi}_h(\vec{r})=\hat{u}(\vec{r})\hat{\Psi}_e^0(\vec{r}),\left[\vec{a}(\vec{r}),\hat{\omega}(\vec{r}^{/})\right]=0.$$

The time derivatives of the dressed field operators can be expressed in terms of derivatives of their components:

$$i\hbar\frac{d}{dt}\hat{\Psi}_e(\vec{r})=i\hbar\frac{d\hat{u}^{+}(\vec{r})}{dt}\hat{\Psi}_e^0(\vec{r})+\hat{u}^{+}(\vec{r})i\hbar\frac{d\hat{\Psi}_e^0(\vec{r})}{dt},$$
$$i\hbar\frac{d}{dt}\hat{\Psi}^h(\vec{r})=i\hbar\frac{d\hat{u}(\vec{r})}{dt}\hat{\Psi}_h^0(\vec{r})+\hat{u}(\vec{r})i\hbar\frac{d\hat{\Psi}_h^0(\vec{r})}{dt}. \tag{56}$$

Using formulas (39), (54), and (55), we obtain

$$i\hbar\frac{d\hat{\Psi}_e(\vec{r})}{dt}=\frac{\hbar^2}{2m_e}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_e(\vec{r})+$$
$$+\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\Psi}_e(\vec{r})+\frac{e^2}{2\hbar c^2}\hat{L}(\vec{r})\hat{\Psi}_e(\vec{r})+\int d^2\vec{r}^{/}V_{Coul.}\left(\vec{r}-\vec{r}^{/}\right)\hat{\rho}\left(\vec{r}^{/}\right)\hat{\Psi}_e(\vec{r}),$$
$$i\hbar\frac{d\hat{\Psi}_h(\vec{r})}{dt}=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_h(\vec{r})-$$
$$-\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\Psi}_h(\vec{r})+\frac{e^2}{2\hbar c^2}\hat{L}(\vec{r})\hat{\Psi}_h(\vec{r})-\int d^2\vec{r}^{/}V_{Coul.}\left(\vec{r}-\vec{r}^{/}\right)\hat{\rho}\left(\vec{r}^{/}\right)\hat{\Psi}_h(\vec{r}), \tag{57}$$
$$\hat{\rho}\left(\vec{r}^{/}\right)=\hat{\rho}_e\left(\vec{r}^{/}\right)-\hat{\rho}_h\left(\vec{r}^{/}\right);\hat{\rho}_i\left(\vec{r}^{/}\right)=\hat{\Psi}_i^{+}\left(\vec{r}^{/}\right)\hat{\Psi}_i\left(\vec{r}^{/}\right),\ i=e,h.$$

Unlike equations of motion (39) for bare field operators $\hat{\Psi}_i^0(\vec{r})$, equations (57) contain new operators, such as $\hat{\omega}(\vec{r})$, $\hat{\omega}(\vec{r})/dt$, and $\vec{\nabla}\hat{\omega}(\vec{r})=\hat{\vec{a}}(\vec{r})$, which characterize the CS gauge field. Similar to the case of a one-component 2DEG, in a two-component 2D e–h system, the quantum point vortices also give rise to vector potential $\vec{a}(\vec{r})$ and scalar potential $d\hat{\omega}/(cdt)$, which appear supplementary to vector potential $\vec{A}(\vec{r})$ of the external magnetic field. However, as noted above, they depend on the difference of the density operators $\hat{\rho}(\vec{r})=\hat{\rho}_e(\vec{r})-\hat{\rho}_h(\vec{r})$ due to different signs of the electrical charges of electrons and holes.

In the case under discussion, vector potential $\vec{a}(\vec{r})$ cannot compensate vector potential $\vec{A}(\vec{r})$ created by an external magnetic field. Chern–Simons vector potential $\vec{a}(\vec{r})$ vanishes in the mean-field approximation, if the average densities $\langle\rho_e\rangle$ and $\langle\rho_h\rangle$ coincide. Nevertheless, numbers ϕ of the quantum point vortices attached to each electron and each hole can be different from zero; in the zero-order approximation, the composite particles will be subjected only to an external magnetic field. In the zero-order approximation, they undergo the Landau quantization under the influence of an external magnetic field and will undergo perturbations in the next orders of the perturbation theory. In addition to the equations of motion (57) characterizing the time evolution of dressed field operators, we need a Hamiltonian describing a 2D e–h system in the presence of the CS gauge field. It can be easily obtained by substituting bare field operators $\hat{\Psi}_i^{0+}(\vec{r})$ and $\hat{\Psi}_i^{0}(\vec{r})$ into Hamiltonian (37) by the dressed field operators

$$\begin{aligned}&\hat{\Psi}_e^0(\vec{r})=u(\vec{r})\hat{\Psi}_e(\vec{r}),\ \hat{\Psi}_h^0(\vec{r})=u^+(\vec{r})\hat{\Psi}_h(\vec{r}),\ \hat{\Psi}_e^{0+}(\vec{r})=\hat{\Psi}_e^+(\vec{r})\hat{u}^+(\vec{r}),\\&\hat{\Psi}_h^{0+}(\vec{r})=\hat{\Psi}_h^+(\vec{r})\hat{u}(\vec{r}),\ \hat{u}(\vec{r})=e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r})},\ \hat{u}^+(\vec{r})=e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r})}.\end{aligned}\tag{58}$$

For example, the first two terms of Hamiltonian (37) are transformed as follows:

$$\begin{aligned}&\hat{K}^0=\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\hat{\Psi}_e^{0+}(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_e^0(\vec{r}^{\,/})+\\&+\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\hat{\Psi}_h^{0+}(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r})\right)^2\hat{\Psi}_h^0(\vec{r}^{\,/})=\\&=\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\hat{\Psi}_e^{+}(\vec{r}^{\,/})e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})\right)^2e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}\hat{\Psi}_e(\vec{r}^{\,/})+\\&+\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\hat{\Psi}_h^{+}(\vec{r}^{\,/})e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})\right)^2e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}\hat{\Psi}_h^0(\vec{r}^{\,/})=\\&=\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\hat{\Psi}_e^{+}(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)^2\hat{\Psi}_e(\vec{r}^{\,/})+\\&+\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\hat{\Psi}_h^{+}(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)^2\hat{\Psi}_h(\vec{r}^{\,/})=\hat{K}\\&\hat{\vec{a}}(\vec{r}^{\,/})=\vec{\nabla}^{/}\hat{\omega}(\vec{r}^{\,/}),\end{aligned}\tag{59}$$

where we use relationships (55):

$$e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)\right)^{2}=\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^{2}e^{-\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})},$$
$$e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)\right)^{2}=\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^{2}e^{\frac{ie}{\hbar c}\hat{\omega}(\vec{r}^{\,/})}. \tag{60}$$

Hamiltonian $\hat{H}^{0}_{Coul}$ of the Coulomb interaction of bare electrons and holes can be transcribed in the dressed field operator representation, taking into account that

$$\hat{\rho}^{0}_{i}\left(\vec{r}\right)=\hat{\Psi}^{0+}_{i}\left(\vec{r}\right)\hat{\Psi}^{0}_{i}\left(\vec{r}\right)=\hat{\rho}\left(\vec{r}\right)=\hat{\Psi}^{+}_{i}\left(\vec{r}\right)\hat{\Psi}_{i}\left(\vec{r}\right),\ i=e,h,$$
$$\hat{H}^{0}_{Coul}=\hat{H}_{Coul}. \tag{61}$$

The Hamiltonian of the e–h system in the presence of the CS gauge field looks as

$$\hat{H}=\hat{K}+\hat{H}_{Coul}=\frac{\hbar^{2}}{2m_{e}}\int d^{2}\vec{r}^{\,/}\hat{\Psi}^{+}_{e}\left(\vec{r}^{\,/}\right)\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^{2}\hat{\Psi}_{e}\left(\vec{r}^{\,/}\right)+$$
$$+\frac{\hbar^{2}}{2m_{h}}\int d^{2}\vec{r}^{\,/}\hat{\Psi}^{+}_{h}\left(\vec{r}^{\,/}\right)\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^{2}\hat{\Psi}_{h}\left(\vec{r}^{\,/}\right)+$$
$$+\frac{1}{2}\int d^{2}\vec{r}^{\,/}\int d^{2}\vec{r}^{\,//}V_{Coul}\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}^{+}_{e}\left(\vec{r}^{\,/}\right)\hat{\rho}_{e}\left(\vec{r}^{\,//}\right)\hat{\Psi}_{e}\left(\vec{r}^{\,/}\right)+$$
$$+\frac{1}{2}\int d^{2}\vec{r}^{\,/}\int d^{2}\vec{r}^{\,//}V_{Coul}\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}^{+}_{h}\left(\vec{r}^{\,/}\right)\hat{\rho}_{h}\left(\vec{r}^{\,//}\right)\hat{\Psi}_{h}\left(\vec{r}^{\,/}\right)-$$
$$-\int d^{2}\vec{r}^{\,/}\int d^{2}\vec{r}^{\,//}V_{Coul}\left(\vec{r}^{\,/}-\vec{r}^{\,//}\right)\hat{\Psi}^{+}_{e}\left(\vec{r}^{\,/}\right)\hat{\rho}_{h}\left(\vec{r}^{\,//}\right)\hat{\Psi}_{e}\left(\vec{r}^{\,/}\right). \tag{62}$$

Hamiltonian (62) is much more complicated than its bare counterpart (37), because it contains a nonlinear form of vector potential operator $\hat{\vec{a}}\left(\vec{r}^{\,/}\right)$ created by the CS gauge field. To obtain equations of motion for dressed field operators, we start with the Schrodinger equations for electrons and holes

$$i\hbar\frac{d\hat{\Psi}_{i}\left(\vec{r}\right)}{dt}=\left[\hat{\Psi}_{i}\left(\vec{r}\right),\hat{H}\right],\ i=e,h. \tag{63}$$

In this case, it is necessary to calculate commutation relations of the field operators with operator $\hat{\vec{a}}\left(\vec{r}^{\,/}\right)$

$$\begin{aligned}
&\left[\hat{\Psi}_e(\vec{r}),\hat{\vec{a}}(\vec{r}^{\,/})\frac{e}{\hbar c}\right]=-\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\hat{\Psi}_e(\vec{r}),\\
&\left[\hat{\Psi}_h(\vec{r}),\hat{\vec{a}}(\vec{r}^{\,/})\frac{e}{\hbar c}\right]=\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\hat{\Psi}_h(\vec{r}),\\
&\left[\hat{\Psi}_e^{+}(\vec{r}),\hat{\vec{a}}(\vec{r}^{\,/})\frac{e}{\hbar c}\right]=\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\hat{\Psi}_e^{+}(\vec{r}),\\
&\left[\hat{\Psi}_h^{+}(\vec{r}),\hat{\vec{a}}(\vec{r}^{\,/})\frac{e}{\hbar c}\right]=-\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\hat{\Psi}_h^{+}(\vec{r}),\\
&\left[\hat{\Psi}_i(\vec{r}),\hat{\vec{a}}(\vec{r})\right]=\left[\hat{\Psi}_i^{+}(\vec{r}),\hat{\vec{a}}(\vec{r})\right]=0,\;\; i=e-h,\\
&\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\Big|_{\vec{r}-\vec{r}^{\,/}}=0.
\end{aligned}\tag{64}$$

Using these equations, we can write

$$\begin{aligned}
&\hat{\Psi}_e(\vec{r})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^2=\\
&=\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)-\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\right)^2\hat{\Psi}_e(\vec{r}),\\
&\hat{\Psi}_h(\vec{r})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^2=\\
&\qquad=\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)-\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\right)^2\hat{\Psi}_h(\vec{r}),\\
&\qquad\hat{\Psi}_e^{+}(\vec{r})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^2=\\
&\qquad=\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)+\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)+\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\right)^2\hat{\Psi}_e^{+}(\vec{r}),\\
&\qquad\hat{\Psi}_h^{+}(\vec{r})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)^2=\\
&\qquad=\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right)-\frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)+\phi\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/}-\vec{r}\right)\right)^2\hat{\Psi}_h^{+}(\vec{r}).
\end{aligned}\tag{65}$$

It should be borne in mind that dressed field operators $\hat{\Psi}_i(\vec{r})$ and $\hat{\Psi}_i^{+}(\vec{r})$ obey the Fermi or Bose statistics with commutation relations

$$\begin{aligned}
&\hat{\Psi}_i(\vec{r})\hat{\Psi}_j^{+}(\vec{r})\pm\hat{\Psi}_j^{+}(\vec{r})\hat{\Psi}_i(\vec{r})=\delta_{ij}\delta^2\left(\vec{r}-\vec{r}^{\,/}\right),\quad (F)\\
&\hat{\Psi}_i(\vec{r})\hat{\Psi}_j^{+}(\vec{r})\pm\hat{\Psi}_j^{+}(\vec{r})\hat{\Psi}_i(\vec{r})=0,\qquad\qquad (B)
\end{aligned}\tag{66}$$

and all the formulas obtained above are valid in both cases. For example, to derive the next commutation relations, we have to use both formulas (65) and (66) as follows:

$$\begin{aligned}
&\left[\hat{\Psi}_e(\vec{r}),\hat{\Psi}_e^+(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)^2\hat{\Psi}_e(\vec{r}^{\,/})\right]=\\
&=\delta^2(\vec{r}-\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)^2\hat{\Psi}_e(\vec{r}^{\,/})+\left(\phi\vec{\nabla}^{/}\theta(\vec{r}^{\,/}-\vec{r})\right)^2\times\\
&\times\hat{\rho}_e(\vec{r}^{\,/})\hat{\Psi}_e(\vec{r}),\\
&\left[\hat{\Psi}_e(\vec{r}),\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\hat{\Psi}_e^+(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)^2\hat{\Psi}_e(\vec{r}^{\,/})\right]=\\
&=\frac{\hbar^2}{2m_e}\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_e(\vec{r})+\\
&+\phi^2\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_e(\vec{r}^{\,/})}{\left|\vec{r}-\vec{r}^{\,/}\right|^2}\hat{\Psi}_e(\vec{r})-\\
&-\phi\frac{\hbar^2}{m_e}\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta(\vec{r}^{\,/}-\vec{r})\hat{\Psi}_e^+(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)\hat{\Psi}_e(\vec{r}^{\,/})\hat{\Psi}_e(r),\\
&\left[\hat{\Psi}_h(\vec{r}),\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\hat{\Psi}_h^+(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)^2\hat{\Psi}_h(\vec{r}^{\,/})\right]=\\
&=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_h(\vec{r})+\\
&+\phi^2\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_h(\vec{r}^{\,/})}{\left|\vec{r}-\vec{r}^{\,/}\right|^2}\hat{\Psi}_h(\vec{r})-\\
&-\phi\frac{\hbar^2}{m_h}\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta(\vec{r}^{\,/}-\vec{r})\hat{\Psi}_h^+(\vec{r}^{\,/})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r}^{\,/})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{\,/})\right)\hat{\Psi}_h(\vec{r}^{\,/})\hat{\Psi}_h(r).
\end{aligned}\tag{67}$$

Here, we took into account the properties described by formulas (64) and the equalities written below

$$\begin{aligned}
&\vec{\nabla}^{/}\vec{A}(\vec{r}^{\,/})=\vec{\nabla}^{/}\hat{\vec{a}}(\vec{r}^{\,/})=0;\ \left[\hat{\Psi}_i(\vec{r}),\hat{\vec{a}}(\vec{r})\right]=0;\\
&\left(\vec{\nabla}^{/}\theta(\vec{r}^{\,/}-\vec{r})\right)^2=\frac{1}{\left|\vec{r}-\vec{r}^{\,/}\right|^2};\ \Delta^{/}\theta(\vec{r}^{\,/}-\vec{r})=0.
\end{aligned}\tag{68}$$

Both integrals proportional to ϕ in equation (67) can be transformed introducing the dressed current density operators for electrons $\hat{\vec{J}}_e(\vec{r})$ and holes $\hat{\vec{J}}_h(\vec{r})$ and the respective continuity equations as follows:

$$\begin{aligned}
&\hat{\vec{J}}_e(\vec{r}) = \frac{\hbar^2}{2m_e i}\left(\hat{\Psi}_e^+(\vec{r})\vec{\nabla}\hat{\Psi}_e(\vec{r}) - \vec{\nabla}\hat{\Psi}_e^+(\vec{r})\hat{\Psi}_e(\vec{r})\right) + \\
&+\left(\frac{e}{m_e c}\vec{A}(\vec{r}) + \frac{e}{m_e c}\hat{\vec{a}}(\vec{r})\right)\hat{\rho}_e(\vec{r}), \\
&\hat{\vec{J}}_h(\vec{r}) = \frac{\hbar^2}{2m_h i}\left(\hat{\Psi}_h^+(\vec{r})\vec{\nabla}\hat{\Psi}_h(\vec{r}) - \vec{\nabla}\hat{\Psi}_h^+(\vec{r})\hat{\Psi}_h(\vec{r})\right) - \\
&-\left(\frac{e}{m_h c}\vec{A}(\vec{r}) + \frac{e}{m_h c}\hat{\vec{a}}(\vec{r})\right)\hat{\rho}_h(\vec{r}). \\
&\frac{d\hat{\rho}_e(\vec{r})}{dt} = -\vec{\nabla}\hat{\vec{J}}_e(\vec{r}), \frac{d\hat{\rho}_h(\vec{r})}{dt} = -\vec{\nabla}\hat{\vec{J}}_h(\vec{r}).
\end{aligned} \tag{69}$$

Condition $\Delta^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right) = 0$ is useful for a simple integral transformation:

$$\begin{aligned}
&-i\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\hat{\Psi}_i^+\left(\vec{r}^{\,/}\right)\vec{\nabla}^{/}\hat{\Psi}_i\left(\vec{r}^{\,/}\right) = \\
&= \frac{i}{2}\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\left(\nabla^{/}\hat{\Psi}_i^+\left(\vec{r}^{\,/}\right)\hat{\Psi}_i\left(\vec{r}^{\,/}\right) - \hat{\Psi}_i^+\left(\vec{r}^{\,/}\right)\vec{\nabla}^{/}\hat{\Psi}_i\left(\vec{r}^{\,/}\right)\right).
\end{aligned}$$

The obtained relations allow us to write the above-mentioned integrals in equations (67) as follows:

$$\begin{aligned}
&\phi\frac{\hbar^2}{m_e}\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\hat{\Psi}_e^+\left(\vec{r}^{\,/}\right)\left(-i\vec{\nabla}^{/} + \frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right) + \frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)\hat{\Psi}_e\left(\vec{r}^{\,/}\right)\hat{\Psi}_e(\vec{r}) = \\
&= \phi\hbar\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\hat{\vec{J}}_e\left(\vec{r}^{\,/}\right)\hat{\Psi}_e\left(\vec{r}^{\,/}\right) = -\phi\hbar\int d^2\vec{r}^{\,/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\times \\
&\times\vec{\nabla}^{/}\hat{\vec{J}}_e\left(\vec{r}^{\,/}\right)\hat{\Psi}_e(\vec{r}) = \phi\hbar\int d^2\vec{r}^{\,/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\frac{d\hat{\rho}_e\left(\vec{r}^{\,/}\right)}{dt}\hat{\Psi}_e(\vec{r}) = -\frac{e}{c}\frac{d\hat{\omega}_e(\vec{r})}{dt}\hat{\Psi}_e(\vec{r}), \\
&\phi\frac{\hbar^2}{m_h}\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\hat{\Psi}_h^+\left(\vec{r}^{\,/}\right)\left(-i\vec{\nabla}^{/} - \frac{e}{\hbar c}\vec{A}\left(\vec{r}^{\,/}\right) - \frac{e}{\hbar c}\hat{\vec{a}}\left(\vec{r}^{\,/}\right)\right)\hat{\Psi}_h\left(\vec{r}^{\,/}\right)\hat{\Psi}_h(\vec{r}) = \\
&= \phi\hbar\int d^2\vec{r}^{\,/}\vec{\nabla}^{/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\hat{\vec{J}}_h\left(\vec{r}^{\,/}\right)\hat{\Psi}_h(\vec{r}) = -\phi\hbar\int d^2\vec{r}^{\,/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\times \\
&\times\vec{\nabla}^{/}\hat{\vec{J}}_h\left(\vec{r}^{\,/}\right)\hat{\Psi}_h(\vec{r}) = \phi\hbar\int d^2\vec{r}^{\,/}\theta\left(\vec{r}^{\,/} - \vec{r}\right)\frac{d\hat{\rho}_h\left(\vec{r}^{\,/}\right)}{dt}\hat{\Psi}_h(\vec{r}) = -\frac{e}{c}\frac{d\hat{\omega}_h(\vec{r})}{dt}\hat{\Psi}_h(\vec{r}), \\
&\hat{\omega}_i(\vec{r}) = -\frac{\phi e}{\alpha}\int d^2\vec{r}^{\,/}\theta\left(\vec{r} - \vec{r}^{\,/}\right)\hat{\rho}_i\left(\vec{r}^{\,/}\right), \quad i = e, h.
\end{aligned} \tag{70}$$

Combining the results expressed by equations (67)–(70), we can formulate the main result of our calculations:

$$\left[\hat{\Psi}_e(\vec{r}),\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{/}\hat{\Psi}_e^+(\vec{r}^{/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{/})\right)^2\hat{\Psi}_e(\vec{r}^{/})\right]=$$
$$=\frac{\hbar^2}{2m_e}\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_e(\vec{r})+\phi^2\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{/}\frac{\hat{\rho}_e(\vec{r}^{/})}{\left|\vec{r}-\vec{r}^{/}\right|^2}\times$$
$$\times\hat{\Psi}_e(\vec{r})+\frac{e}{c}\frac{d\hat{\omega}_e(\vec{r})}{dt}\hat{\Psi}_e(\vec{r}),$$
$$\left[\hat{\Psi}_h(\vec{r}),\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{/}\hat{\Psi}_h^+(\vec{r}^{/})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r}^{/})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{/})\right)^2\hat{\Psi}_h(\vec{r}^{/})\right]=$$
$$=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r})\right)^2\hat{\Psi}_h(\vec{r})+\phi^2\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{/}\frac{\hat{\rho}_h(\vec{r}^{/})}{\left|\vec{r}-\vec{r}^{/}\right|^2}\times \tag{71}$$
$$\times\hat{\Psi}_h(\vec{r})+\frac{e}{c}\frac{d\hat{\omega}_h(\vec{r})}{dt}\hat{\Psi}_h(\vec{r}).$$

In a similar way, the following commutations can be derived:

$$\left[\hat{\Psi}_e(\vec{r}),\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{/}\hat{\Psi}_h^+(\vec{r}^{/})\left(-i\vec{\nabla}^{/}-\frac{e}{\hbar c}\vec{A}(\vec{r}^{/})-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{/})\right)^2\hat{\Psi}_h(\vec{r}^{/})\right]=$$
$$=\phi^2\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{/}\frac{\hat{\rho}_h(\vec{r}^{/})}{\left|\vec{r}-\vec{r}^{/}\right|^2}\hat{\Psi}_e(\vec{r})-\frac{e}{c}\frac{d\hat{\omega}_h(\vec{r})}{dt}\hat{\Psi}_e(\vec{r}),$$
$$\left[\hat{\Psi}_h(\vec{r}),\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{/}\hat{\Psi}_h^+(\vec{r}^{/})\left(-i\vec{\nabla}^{/}+\frac{e}{\hbar c}\vec{A}(\vec{r}^{/})+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}^{/})\right)^2\hat{\Psi}_e(\vec{r}^{/})\right]= \tag{72}$$
$$=\phi^2\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{/}\frac{\hat{\rho}_e(\vec{r}^{/})}{\left|\vec{r}-\vec{r}^{/}\right|^2}\hat{\Psi}_h(\vec{r})-\frac{e}{c}\frac{d\hat{\omega}_e(\vec{r})}{dt}\hat{\Psi}_h(\vec{r}).$$

Formulas (71) and (72) constitute the basis for the main statements:

$$
\begin{aligned}
&\left[\hat{\Psi}_e(\vec{r}),\hat{K}\right]=\frac{\hbar^2}{2m_e}\left(-i\vec{\nabla}+\frac{e}{\hbar c}\vec{A}(\vec{r}\ )+\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}\ )\right)^2\hat{\Psi}_e(\vec{r}\ )+\\
&+\phi^2\hat{L}(\vec{r})\hat{\Psi}_e(\vec{r})+\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\Psi}_e(\vec{r}),\\
&\left[\hat{\Psi}_h(\vec{r}),\hat{K}\right]=\frac{\hbar^2}{2m_h}\left(-i\vec{\nabla}-\frac{e}{\hbar c}\vec{A}(\vec{r}\ )-\frac{e}{\hbar c}\hat{\vec{a}}(\vec{r}\ )\right)^2\hat{\Psi}_h(\vec{r}\ )+\\
&+\phi^2\hat{L}(\vec{r})\hat{\Psi}_h(\vec{r})-\frac{e}{c}\frac{d\hat{\omega}(\vec{r})}{dt}\hat{\Psi}_h(\vec{r}),\\
&\hat{L}(\vec{r})=\frac{\hbar^2}{2m_e}\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_e(\vec{r}^{\,/})}{\left|\vec{r}-\vec{r}^{\,/}\right|^2}+\frac{\hbar^2}{2m_h}\int d^2\vec{r}^{\,/}\frac{\hat{\rho}_h(\vec{r}^{\,/})}{\left|\vec{r}-\vec{r}^{\,/}\right|^2},\\
&\hat{\omega}(\vec{r})=\hat{\omega}_e(\vec{r})-\hat{\omega}_h(\vec{r}),\hat{\omega}_i(\vec{r})=-\frac{\phi e}{\alpha}\int d^2\vec{r}^{\,/}\theta(\vec{r}-\vec{r}^{\,/})\hat{\rho}_i(\vec{r}^{\,/}).
\end{aligned}
\tag{73}
$$

The obtained results confirm the earlier derived equations of motion (57) for dressed field operators.

4. Influence of the CS Gauge Field on the Energy Level of a Two-Dimensional Magnetoexciton

In the Landau gauge description, two-dimensional electrons and holes are described as free particles moving along the in-plane x-axis and undergoing the Landau quantization along the in-plane *y*-axis perpendicular to the *x*-axis. The free motion is represented by the plane wave functions with unidimensional (1D) wave vectors p and q as quantum numbers, whereas the Landau quantization takes place in the form of harmonic oscillations around the gyration points situated on the *y*-axis at distances pl_0^2 and $-ql_0^2$ from the origin, where l_0 is the magnetic length. The displacements of electrons and holes in the opposite parts of the *y*-axis are due to the different signs of their electric charges. The single particle wave functions corresponding to the lowest energy level of the Landau quantization are as follows:

$$
\begin{aligned}
&\varphi_e(\vec{r})=\frac{e^{ipx}}{\sqrt{L_x l_0\sqrt{\pi}}}\exp\left[-\frac{\left(y-pl_0^2\right)^2}{2l_0^2}\right],\\
&\varphi_h(\vec{r})=\frac{e^{iqx}}{\sqrt{L_x l_0\sqrt{\pi}}}\exp\left[-\frac{\left(y+ql_0^2\right)^2}{2l_0^2}\right].
\end{aligned}
\tag{74}
$$

Here, we consider the 2D layer with a surface area of $S=L_xL_y$. Wave functions (74) belong to the lowest levels of Landau quantization with quantum numbers $n_e=n_h=0$; in this paper, we do not consider the excited Landau quantization energy levels. This approach is known as the lowest Landau Levels (LLL) approximation and labels n_e and n_h at electron field operators a_p^+, a_p and hole field operators b_p^+, b_p will be dropped. Bare electron and hole field operators obey the Fermi statistics. The energies of the electrons and holes are $\hbar\omega_{c,i}/2$, with $i = e, h$, where $\hbar\omega_{c,i}$ are the cyclotron frequencies. The electron and hole field operators in the coordinate representation in the LLL–approximation are as follows:

$$
\begin{aligned}
\hat{\Psi}_e(\vec{r}) &= \frac{1}{\sqrt{L_x l_0 \sqrt{\pi}}} \sum_p e^{ipx} \exp\left[-\frac{\left(y - p l_0^2\right)^2}{2 l_0^2}\right] a_p, \\
\hat{\Psi}_h(\vec{r}) &= \frac{1}{\sqrt{L_x l_0 \sqrt{\pi}}} \sum_q e^{iqx} \exp\left[-\frac{\left(y + q l_0^2\right)^2}{2 l_0^2}\right] b_q,
\end{aligned} \tag{75}
$$

where we introduced the electron and hole density operators

$$
\begin{aligned}
\hat{\rho}_e(\vec{r}) &= \hat{\Psi}_e^+(\vec{r})\hat{\Psi}_e(\vec{r}) = \frac{1}{L_x l_0 \sqrt{\pi}} \sum_{p,q} e^{i(p-q)x} \exp\left[-\frac{\left(y - p l_0^2\right)^2}{2 l_0^2} - \frac{\left(y - q l_0^2\right)^2}{2 l_0^2}\right] a_q^+ a_p, \\
\hat{\rho}_h(\vec{r}) &= \hat{\Psi}_h^+(\vec{r})\hat{\Psi}_h(\vec{r}) = \frac{1}{L_x l_0 \sqrt{\pi}} \sum_{\mathrm{u,v}} e^{i(u-v)x} \exp\left[-\frac{\left(y + u l_0^2\right)^2}{2 l_0^2} - \frac{\left(y + \mathrm{v} l_0^2\right)^2}{2 l_0^2}\right] b_{\mathrm{v}}^+ b_u, \\
\hat{\rho}(\vec{r}) &= \hat{\rho}_e(\vec{r}) - \hat{\rho}_h(\vec{r}).
\end{aligned} \tag{76}
$$

Here, the density operator of the e–h system $\hat{\rho}(\vec{r})$ was defined as the algebraic sum of the electron and hole density operators. This algebraic sum determines the CS gauge field vector potential operator $\hat{\vec{a}}(\vec{r})$ as follows:

$$
\begin{aligned}
&\hat{\vec{a}}(\vec{r}) = -\frac{\phi e}{\alpha} \int d^2\vec{r}^{\,\prime} \vec{\nabla}_{\vec{r}} \theta\left(\vec{r} - \vec{r}^{\,\prime}\right) \hat{\rho}\left(\vec{r}^{\,\prime}\right), \\
&\frac{e^2}{2 m_e c^2} \hat{\vec{a}}^2(\vec{r}) = -\frac{\hbar^2 \phi^2}{2 m_e} \int d^2\vec{r}^{\,\prime} \int d^2\vec{r}^{\,\prime\prime} \frac{\left(\vec{r} - \vec{r}^{\,\prime}\right)\left(\vec{r} - \vec{r}^{\,\prime\prime}\right)}{\left|\vec{r} - \vec{r}^{\,\prime}\right|^2 \left|\vec{r} - \vec{r}^{\,\prime\prime}\right|^2} \hat{\rho}\left(\vec{r}^{\,\prime}\right) \hat{\rho}\left(\vec{r}^{\,\prime\prime}\right), \\
&\theta\left(\vec{r} - \vec{r}^{\,\prime}\right) = \arctan\left(\frac{y - y^{\prime}}{x - x^{\prime}}\right); \\
&\vec{\nabla}_{\vec{r}} = \vec{e}_x \frac{\partial}{\partial x} + \vec{e}_y \frac{\partial}{\partial x}, \\
&\vec{\nabla}_{\vec{r}} \theta\left(\vec{r} - \vec{r}^{\,\prime}\right) = \frac{-\vec{e}_x\left(y - y^{\prime}\right) + \vec{e}_y\left(x - x^{\prime}\right)}{\left|\vec{r} - \vec{r}^{\,\prime}\right|^2}, \ \alpha = \frac{e^2}{\hbar c} = \frac{1}{137}.
\end{aligned} \tag{77}
$$

To determine the influence of the CS gauge field on the 2D magnetoexciton energy level, it is necessary to calculate the average value of Hamiltonian (62) using magnetoexciton wave function $\left|\Psi_{ex}\left(\vec{k}\right)\right\rangle$, which was obtained in [13]:

$$\left|\Psi_{ex}\left(\vec{k}\right)\right\rangle = \Psi_{ex}^{+}\left(\vec{k}\right)|0\rangle,$$
$$\Psi_{ex}^{+}\left(\vec{k}\right) = \frac{1}{\sqrt{N}}\sum_{t} e^{ik_y t l_0^2} a^{+}_{t+\frac{k_x}{2}} b^{+}_{-t+\frac{k_x}{2}}, \tag{78}$$
$$N = \frac{S}{2\pi l_0^2}.$$

First, we will discuss the influence of the terms in Hamiltonian (62), which are proportional to the square of CS vector potential $\hat{\vec{a}}\left(\vec{r}\right)$ in the form

$$\frac{e^2}{2m_{e,h}c^2}\int d^2\vec{r}^{\,/}\hat{\Psi}_{e,h}^{+}(\vec{r})\hat{\vec{a}}^2\left(\vec{r}\right)\hat{\Psi}_{e,h}\left(\vec{r}\right). \tag{79}$$

Their average value calculated with the magnetoexciton wave function with wave vector $\vec{k} = 0$ is as follows:

$$\begin{aligned}
\Delta_e &= \left\langle\Psi_{ex}\left(0\right)\right|\frac{e}{2m_e c^2}\int d^2\vec{r}\hat{\Psi}_e^{+}\left(\vec{r}\right)\hat{\vec{a}}^2\left(\vec{r}\right)\hat{\Psi}_e\left(\vec{r}\right)\left|\Psi_{ex}\left(0\right)\right\rangle = \\
&= \frac{\hbar^2\phi^2}{m_e l_0^2}\cdot\frac{1}{8\pi^2}\int d^2\vec{\rho}_1\int d^2\vec{\rho}_2\,\frac{\vec{\rho}_1\cdot\vec{\rho}_2}{\vec{\rho}_1^{\,2}\cdot\vec{\rho}_2^{\,2}}\exp\left\{-\frac{1}{4}\left(\vec{\rho}_1^{\,2}+\vec{\rho}_2^{\,2}+\left|\vec{\rho}_1-\vec{\rho}_2\right|^2\right)+\right. \\
&\left.+\frac{i}{2}\left[\vec{\rho}_2\times\vec{\rho}_1\right]\right\} = \frac{\hbar^2\phi^2}{m_e l_0^2}\cdot\frac{1}{8\pi^2}\int_0^{\infty}d\rho_1\int_0^{\infty}d\rho_2\int_0^{2\pi}d\varphi_1\int_0^{2\pi}d\varphi_2\cos\left(\varphi_1-\varphi_2\right)\times \\
&\times\exp\left\{-\frac{1}{2}\left[\rho_1^2+\rho_2^2-\rho_1\rho_2\cos\left(\varphi_1-\varphi_2\right)\right]+\frac{i}{2}\rho_2\rho_1\sin\left(\varphi_1-\varphi_2\right)\right\}.
\end{aligned} \tag{80}$$

The Fourier series expansions of exponents $\exp\left(iz\sin t\right)$ and $\exp\left(z\cos t\right)$ contain coefficients expressed in terms of Bessel functions $J_\nu\left(z\right)$ and modified Bessel functions $I_\nu\left(z\right)$ [14]:

$$\begin{aligned}
e^{iz\sin t} &= J_0\left(z\right)+2\sum_{k=1}^{\infty}\left[J_{2k}\left(z\right)\cos 2kt + iJ_{2k-1}\left(z\right)\sin\left(2k-1\right)t\right], \\
e^{z\cos t} &= I_0\left(z\right)+2\sum_{k=1}^{\infty}I_k\left(z\right)\cos kt.
\end{aligned} \tag{81}$$

Substitution of them into the previous expression leads to its transformation

$$\Delta_e = \frac{\hbar^2\phi^2}{m_e l_0^2}\left\{\frac{1}{4\pi^2}\int_0^\infty d\rho_1\int_0^\infty d\rho_2 \exp\left[-\frac{1}{2}\left(\rho_1^2+\rho_2^2\right)\right]J_0\left(\frac{1}{2}\rho_1\cdot\rho_2\right)\times\right.$$
$$\times I_1\left(\frac{1}{2}\rho_1\cdot\rho_2\right)+\frac{1}{2}\sum_{m=1}^\infty\int_0^\infty d\rho_1\int_0^\infty d\rho_2 \exp\left[-\frac{1}{2}\left(\rho_1^2+\rho_2^2\right)\right]J_{2m}\left(\frac{1}{2}\rho_1\cdot\rho_2\right)\times \tag{82}$$
$$\left.\times\left[I_{2m+1}\left(\frac{1}{2}\rho_1\cdot\rho_2\right)+I_{2m-1}\left(\frac{1}{2}\rho_1\cdot\rho_2\right)\right]\right\}.$$

In subsequent calculations, we will use integrals with two Bessel functions [14]:

$$\int_0^\infty x^{\alpha-1}e^{-px^2}J_\mu(bx)I_\nu(cx)dx = \frac{b^\mu c^\nu (p)^{-\frac{(\alpha+\mu+\nu)}{2}}}{2^{\mu+\nu+1}\Gamma(\mu+1)}\times$$
$$\times\sum_{k=0}^\infty\frac{1}{k!}\Gamma\begin{bmatrix}k+\frac{(\alpha+\mu+\nu)}{2}\\ \nu+k+1\end{bmatrix}\left(\frac{c}{2p}\right)^{2k}\cdot{}_2F_1\left(-k,-(\nu+k),\mu+1,-\frac{b^2}{c^2}\right). \tag{83}$$

Their applications give rise to energy shift Δ_e of the magnetoexciton energy level

$$\Delta_e = \frac{\hbar^2\phi^2}{m_e l_0^2}\left\{\frac{1}{16\pi^2}\sum_{k=0}^\infty\frac{1}{2^k(k+1)}\times\right.$$
$$\times{}_2F_1(-k,-(k+1),1,-1)+\frac{1}{2}\sum_{m=1}^\infty\sum_{k=0}^\infty\frac{(2m+k)!}{2^{4m+k+2}\Gamma(2m+1)}\times$$
$$\times\frac{1}{k!(k+2m+1)}\cdot{}_2F_1(-k,-(k+2m+1),2\mathrm{m}+1,-1)+ \tag{84}$$
$$+\frac{1}{2}\sum_{m=1}^\infty\sum_{k=0}^\infty\frac{(2m+k-1)!}{2^{4m+k}\Gamma(2m+1)}\frac{1}{k!}\times$$
$$\left.\times{}_2F_1(-k,-(k+2m-1),2\mathrm{m}+1,-1)\right\}.$$

It is possible to estimate, along with terms (79) containing the square of the CS vector potential $\hat{\vec{a}}^2(\vec{r})$, the contribution of the mixed term proportional to the scalar product of two vector potentials $\hat{\vec{a}}(\vec{r})$ and $\vec{A}(\vec{r})$. It is expressed by the average value

$$\left\langle\Psi_{ex}(\vec{k})\right|\frac{e^2}{m_e c^2}\int d^2\vec{r}^{\,/}\hat{\Psi}_e^+(\vec{r}^{\,/})\hat{\vec{a}}^2(\vec{r}^{\,/})\vec{A}(\vec{r}^{\,/})\hat{\Psi}_e(\vec{r}^{\,/})\left|\Psi_{ex}(\vec{k})\right\rangle=$$
$$=\frac{\hbar eB\phi}{4\pi^2 m_e c l_0^4 N}\cdot\int d^2\vec{r}^{\,/}\int d^2\vec{r}^{\,//}\frac{y^/(y^/-y^{//})}{\left|\vec{r}^{\,/}-\vec{r}^{\,//}\right|^2}\exp\left[-\frac{1}{2}\left|\frac{\vec{r}^{\,/}-\vec{r}^{\,//}}{l_0}\right|^2-\right. \tag{85}$$
$$\left.-\frac{(\vec{k}l_0)^2}{2}+(y^/-y^{//})k_x-(x^/-x^{//})k_y\right].$$

The shift of the magnetoexciton energy level $\Delta_e^{/}$ at point $\vec{k}=0$ due this term can be calculated exactly:

$$\Delta_e^{/}=\frac{\hbar eB\phi}{4m_e c}. \tag{86}$$

In the case of electron effective mass m_e equal to free electron mass m_0 at a magnetic field strength of $B=10$ T and $\phi=1$, the shift of the magnetoexciton energy level at point $\vec{k}=0$ due to the influence of the CS gauge field can be estimated as $\Delta_e^{/}=1/4$ meV .

5. Conclusions

The origin of the CS gauge field, as well as quantum point vortices, is associated with a collective motion in the 2D system, where the main role is played by angles $\theta\left(\vec{r}-\vec{r}^{/}\right)$ created by reference vectors $\left(\vec{r}-\vec{r}^{/}\right)$ with a selected axis. The reference vectors describe positions of the particles at points $\vec{r}^{/}$ with density operator $\hat{\rho}\left(\vec{r}^{/}\right)$. The coherent summation of the angles weighted with the density operator gives rise to phase operator $\hat{\omega}\left(\vec{r}\right)$, whereas the gradients of the angles and their weighted summation give rise to vector potential $\hat{\vec{a}}\left(\vec{r}\right)$ of the CS gauge field.

Unitary transformation operators $\exp\left(\pm ie\hat{\omega}(\vec{r})/(\hbar c)\right)$ acting on bare electron and hole field operators lead to the formation of dressed field operators representing composite particles with number ϕ of attached quantum point vortices. Dressed field operators obey the Fermi or Bose statistics depending on the parity of numbers ϕ of attached vortices. A Hamiltonian describing the composite particles and their interactions through the Coulomb forces and under the influence of the CS gauge field has been deduced; equations of motion for dressed field operators have been derived. The influence of the CS gauge field on the energy levels of 2D magnetoexcitons has been estimated.

References

[1] S. S. Chern and J. Simons, Ann. Math. 99, 48 (1974).
[2] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
[3] N. Read, Phys. Rev. B 58, 16262 (1998).
[4] F. Wilczek, Phys. Rev. Lett. 48, 1144, (1982); 49, 957 (1982).
[5] S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. 62, 82 (1989).
[6] J. K. Jain, Phys. Rev. Lett. 63, 199 (1989).
[7] G. Dunne, R. Jackiw, and C. Trugenberger, Phys. Rev. D 41, 661 (1990).
[8] R. Jackiw and So Young Pi, Phys. Rev. D 42, 3500 (1990).
[9] S. A. Moskalenko and V. A. Moskalenko, Mold. J. Phys. Sci. 16 (3–4), 133 (2017).
[10] S. A. Moskalenko, V. A. Moskalenko, D. F. Digor, and I. A. Lelyakov, Mold. J. Phys. Sci. 16 (3–4), 173 (2017).
[11] S. A. Moskalenko and V. A. Moskalenko, Mold. J. Phys. Sci. 17 (1–2), 41 (2018).

[12] S. A. Moskalenko, V. A. Moskalenko, I. V. Podlesny, and I. A. Zubac, Semiconductors 53, 2055 (2019).
[13] S. A. Moskalenko, V. A. Moskalenko, P. I. Khadzhi, I. V. Podlesny, M. A. Liberman, and I. A. Zubac, Mold. J. Phys. Sci. 17, 52 (2018).
[14] A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marichev, Integrals and Series: Special Functions, Gordon and Breach Science Publishers, New York, 1986.