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POSSIBLE STRONG GRAVITATIONAL WAVE SOURCES FOR THE *LISA* ANTENNA

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ABSTRACT

Recently G. M. Fuller and X. Shi proposed that the gravitational collapse of supermassive objects ($M \gtrsim 10^4\ M_\odot$) could be a cosmological source of γ-ray bursts (GRBs). The major advantage of their model is that supermassive object collapses are far more energetic than solar mass–scale compact mergers. Also, in their proposal the seeds of supermassive black holes (SMBHs) thus formed could give rise to the SMBHs observed at the center of many galaxies. We argue here that, besides the generation of GRBs, there could well occur a strong generation of gravitational waves (GWs) during the formation of SMBHs. As a result, the rate of such GW bursts could be as high as the rate of GRBs in the model by G. M. Fuller and X. Shi. In this case, the detection of GRBs and bursts of GWs should occur with a small time difference. We also argue that the GWs produced by the SMBHs studied here could be detected when the *Laser Interferometric Space Antenna* becomes operative.

Subject headings: black hole physics — gamma rays: bursts

1. INTRODUCTION

The *Laser Interferometric Space Antenna* (*LISA*) is designed to detect low-frequency gravitational waves in the frequency range 10^{-4}–1 Hz, which cannot be detected on the Earth because of seismic noise. A lot of very interesting astrophysical phenomena are believed to generate gravitational waves (GWs) in this frequency band: the formation of supermassive black holes (SMBHs), SMBH-SMBH binary coalescence, compact stars orbiting SMBHs in galactic nuclei, pairs of close white dwarfs, pairs of neutron stars, neutron star and black hole binaries, pairs of contact normal stars, normal star and white dwarf binaries, and pairs of stellar black holes.

We are particularly concerned here with SMBHs, which are believed to be present in galactic nuclei (Blandford 1999). Lynden-Bell (1969) originally proposed that active galaxies harbor a SMBH engine fed by accretion and there is now solid observational evidence for this (Richstone et al. 1998), although there remain some unanswered questions related to their formation. Several interesting papers study the mass function of SMBHs in galaxies (Franceschini, Vercellone, & Fabian 1998; Salucci et al. 1999), using different combinations of optical, infrared, radio, and X-ray data.

SMBHs could form through the dynamical evolution of dense star cluster objects by the merging of SMBHs of smaller masses and by the viscous evolution and collapse of self-gravitating gaseous objects (e.g., supermassive stars). Quinlan & Shapiro (1990) assumed the existence of a dense star cluster in a galactic nucleus and followed the buildup of 100 $M_\odot$ or larger seed black holes by collisions. Another possibility is that $\sim 10^6\ M_\odot$ SMBHs form by coherent collapse in galactic nuclei before most of the bulge gas turns into stars (Silk & Rees 1998; Haehnelt, Natarajan, & Rees 1998). Other interesting studies concerning SMBH formation are discussed by Rees (1997, 1998), Haehnelt & Rees (1993), Haehnelt (1994), Eisenstein & Loeb (1995), Umemura, Loeb, & Turner (1993), and Fuller & Shi (1998, hereafter FS).

SMBHs may produce a strong GW signal during their formation, which could be detectable by *LISA* even at cosmological distances. Since most galaxies could harbor SMBHs it is argued that the number of events expected could be several per year or even per day.

It is worth studying whether other astrophysical phenomena related to the formation of such putative SMBHs, such as the emission of electromagnetic radiation and neutrinos, could help constrain the SMBH production rate and formation epoch. For example, γ-ray bursts (GRBs) could be related to the production of GWs since the formation of SMBHs may be a very energetic phenomenon. In particular, GRBs have been puzzling astrophysicists because of the enormous electromagnetic energy produced, $\sim 10^{51}$–10^{52} ergs, the spatial isotropy (which suggests that the sources are cosmological), and the event rate of several sources per day.

Recently FS (see also Shi & Fuller 1998; Abazajian, Fuller, & Shi 1999) proposed that the gravitational collapse of supermassive objects ($M \gtrsim 10^4\ M_\odot$), either as relativistic star clusters or as a single supermassive star could account for cosmological GRBs. These authors also proposed that such supermassive objects should produce neutrino emission, but they did not consider whether such γ-ray and neutrino sources could be also strong GW sources. Since the FS model involves the formation of a SMBH it is hard to avoid GWs being also produced.

The paper is organized as follows: § 2 deals with the GWs generated by GRB SMBHs and § 3 presents the discussion and conclusions.

2. GRAVITATIONAL WAVES FROM GRB SMBHs

This paper extends the study by FS, which considers whether the collapse of supermassive objects could account for cosmological GRBs. We argue that such a source of γ-rays could also be a strong source of GWs. Then we propose an independent way to check FS model through GW astronomy.

FS define a supermassive object in terms of a star or star cluster that undergoes the general relativistic Feynman-Chandrasekhar instability during its evolution. Supermassive objects with $M \gtrsim 5 \times 10^4\ M_\odot$ could leave black hole remnants of $M \gtrsim 10^3\ M_\odot$. To account for the

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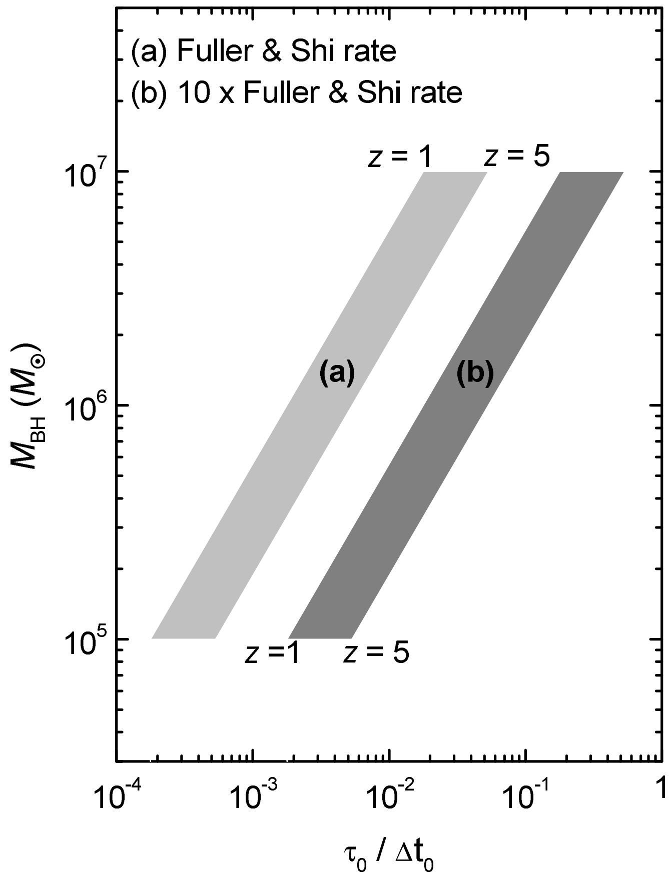

FIG. 1.—Duty cycle vs. the mass of SMBHs for the formation redshift range $z = 1$–5. The results are presented for 1 and 10 events day^{-1}. The cosmological model considered has $\Omega_0 = \Omega_b = 0.1$ and $H_0 = 50$ km s^{-1} Mpc^{-1}.

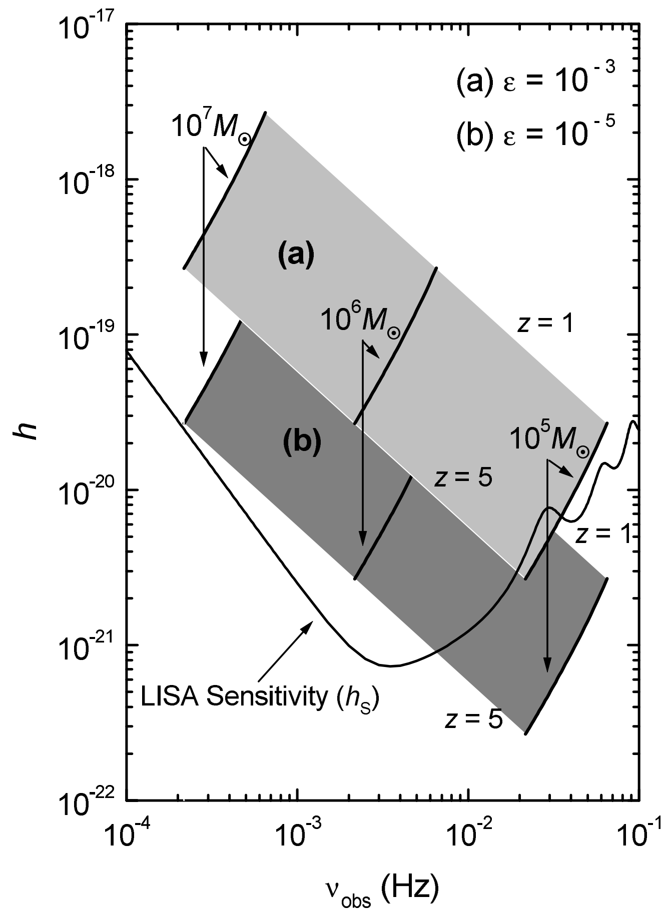

FIG. 2.—Dimensionless amplitude $h_{\rm BH}$ as a function of observed frequency for $\varepsilon = 10^{-5}$ and $\varepsilon = 10^{-3}$ for the burst of GWs for $M_{\rm BH} = 10^5$, 10^6, and 10^7 $M_\odot$ at redshifts $z = 1$–5. The *LISA* sensitivity for burst sources (h_S) is also plotted. The cosmological model considered has $\Omega_0 = \Omega_b = 0.1$ and $H_0 = 50$ km s^{-1} Mpc^{-1}.

observed rate of GRBs the supermassive object collapses should amount to several per day. Each collapse probably leads to a black hole remnant, so it is hard to avoid the conclusion that GWs are generated with the same frequency. If other processes of SMBH formation do not involve GRB events, the GW production rate could well be even higher.

If all supermassive objects form and collapse at a redshift z, as assumed by FS, the event rate is

$$R_{\rm BH} \simeq 4\pi r^2 a_z^3 \frac{dr}{dt_0} \frac{\rho_b F(1+z)^3}{M} , \quad (1)$$

where r is the Friedmann-Robertson-Walker comoving coordinate of the supermassive object, a_z is scale factor of the universe at redshift z, t_0 the age of the universe, ρ_b is the present value of the baryonic density, F is the fraction of baryons incorporated in supermassive objects, and M is the mass of the initial hydrostatic supermassive star, taken to be $M = 10M_{\rm BH}$, where $M_{\rm BH}$ is the mass of the resulting SMBH (FS; Shi & Fuller 1998; Abazajian et al. 1999). This rate can be rewritten as

$$R_{\rm BH} \simeq 4\pi r^2 c n_{\rm BH} , \quad (2)$$

where $n_{\rm BH}$ is the number density of SMBHs, given by

$$n_{\rm BH} = \frac{\rho_b F}{M} . \quad (3)$$

Equation (2) is implicit in the equations derived by Carr (1980) in a study concerning the generation of GWs from SMBHs.

The GW amplitude associated with the formation of each SMBH is (Thorne 1987)

$$h_{\rm BH} = \left(\frac{15}{2\pi}\varepsilon\right)^{1/2} \frac{G}{c^2}\frac{M_{\rm BH}}{r_0}$$
$$\simeq 7.4 \times 10^{-20} \varepsilon^{1/2} \left(\frac{M_{\rm BH}}{M_\odot}\right)\left(\frac{r_0}{1\ {\rm Mpc}}\right)^{-1} , \quad (4)$$

where ε is the efficiency of generation of GWs. The collapse to a black hole produces a signal with frequency

$$\nu_{\rm obs} = \frac{1}{5\pi M_{\rm BH}} \frac{c^3}{G} (1+z)^{-1}$$
$$\simeq 1.3 \times 10^4 {\rm Hz} \left(\frac{M_\odot}{M_{\rm BH}}\right)(1+z)^{-1} . \quad (5)$$

The ensemble of SMBHs formed should produce a background of GWs with amplitude

$$h_{\rm BG}^2 = \frac{1}{\nu_{\rm obs}} \int h_{\rm BH}^2 \, dR_{\rm BH} \quad (6)$$

(de Araujo, Miranda, & Aguiar 2000; O. D. Miranda et al. 2000, in preparation), where $dR_{\rm BH}$ is the differential SMBH formation rate. If the SMBHs are assumed to have the same mass and formation redshift, as in the FS model, we have

$$h_{\rm BG} = \left(\frac{4\pi R^2 c n_{\rm BH}}{\nu_{\rm obs}}\right)^{1/2} h_{\rm BH} . \quad (7)$$

This equation can be written as

$$h_{\rm BG} = \left(\frac{\tau}{\Delta t}\right)_0^{1/2} h_{\rm BH} \quad (8)$$

(see Carr 1980), where the subscript zero indicates a present-day value, τ_0 is the duration of each burst, and Δt_0 is the interval between bursts. Unlike Carr, we assume that the above equation holds only for $(\tau/\Delta t)_0 \gtrsim 1$. These timescales are

$$\tau_0 \simeq \frac{1}{\nu_{\rm obs}} , \quad (9)$$

and

$$\Delta t_0 \simeq \frac{1}{R_{\rm BH}} . \quad (10)$$

The ratio

$$\left(\frac{\tau}{\Delta t}\right)_0 \simeq \frac{4\pi R^2 c n_{\rm BH}}{\nu_{\rm obs}} \quad (11)$$

is called duty cycle and can be interpreted as the number of overlapping bursts.

If the bursts overlap, $(\tau/\Delta t)_0$ is greater than 1 and thus $h_{\rm BG} > h_{\rm BH}$; on the other hand, if $(\tau/\Delta t)_0$ is less than 1, they do not overlap and the GW background is not continuous, but consists of a sequence of spaced bursts with a mean separation $\sim \Delta t_0$ (see Ferrari, Matarrese, & Schneider 1999, who consider the case where a noncontinuous background also appears).

The cosmological model considered here has a density parameter $\Omega_0 = \Omega_b = 0.1$ and Hubble constant $H_0 = 50$ km s^{-1} Mpc^{-1}. For a SMBH formed at redshift $z \simeq 3$ with mass $10^7\ M_\odot$, the GWs would be detected at frequency $\nu_{\rm obs} \simeq 3.3 \times 10^{-4}$ Hz, so the characteristic duration of the burst is $\tau_0 \simeq 3 \times 10^3$ s. If $\Delta t_0 \simeq 1/R_{\rm BH} = 1$ day^{-1}, as observed for GRBs, we obtain 4.0×10^{-2} for the duty cycle. In this case, a population of SMBHs formed at $z \simeq 3$ with mass $10^7\ M_\odot$ cannot produce a background and one will observe a burst a day with duration τ_0, amplitude $h_{\rm BH}$, and frequency $\nu_{\rm obs}$.

The results are summarized in Figure 1 which shows the duty cycle $(\tau_0/\Delta t_0)$ as a function of the mass of the SMBHs, for the formation redshift range $z = 1$–5. We also present, for comparison, results for $R_{\rm BH} \sim 10$ day^{-1}.

The energy density of the GWs can be written in units of the critical density as

$$\Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d \log \nu_{\rm obs}} , \quad (12)$$

where $\rho_c = 3H^2/8\pi G$. Equivalently

$$\Omega_{\rm GW} = \frac{\nu_{\rm obs}}{c^3 \rho_c} F_\nu = \frac{4\pi^2}{3H^2}\, \nu_{\rm obs}^2 h_{\rm BH}^2 . \quad (13)$$

Assuming a maximum efficiency for the generation of GWs ($\varepsilon \simeq 7 \times 10^{-4}$; Stark & Piran 1986) during the collapse of an object to a black hole, one has $\Omega_{\rm GW} < 10^{-6}$ for the redshifts and masses studied here.

In Figure 2 we present the amplitude $h_{\rm BH}$ as a function of the observed frequency ($\nu_{\rm obs}$) for different values of ε, SMBH mass and formation redshift. We also present the *LISA* sensitivity (h_s) for a signal-to-noise ratio of 1 for burst sources.

For example, $h_{\rm BH} > h_s$ for $M_{\rm BH} = 10^6\ M_\odot$ and $\varepsilon > 10^{-5}$. Thus, even for low GW efficiency the signal produced by these SMBHs could be detected by *LISA*.

3. DISCUSSION AND CONCLUSIONS

The results presented here were obtained for an open universe model with $\Omega_b = 0.1$ and $H_0 = 50$ km s^{-1} Mpc^{-1}. We also assume the same scenario as FS, with all the SMBHs forming at the same redshift. For a given event rate, and for a given range of mass, we first calculate the duty cycle to see whether the GWs produced by the ensemble of SMBHs generate a stochastic background. For an event rate exceeding 1–10 day^{-1} we find that the bursts do not overlap and so they do not produce a continuous stochastic background. In particular, a stochastic background could occur for black holes with $M_{\rm BH} \sim 10^7\ M_\odot$ only if the event rate exceeded 30 day^{-1}. In this case we would have $\tau_0/\Delta t_0 > 1$ and the GWs of different seeds could overlap producing a background with amplitude given by equation (7). SMBHs formed with masses less than $10^6\ M_\odot$ could produce a GW background for the same event rate only if they formed at $z > 5$.

The major advantage of the FS scenario, as a cosmological source of γ-ray emission, is its enormous energy reservoir; the gravitational binding energy is $E_g \sim 10^{54}$ $(M_{\rm BH}/M_\odot)$ ergs. Another advantage of this scenario is related to the angular scale of the sources. Although tremendous energy is deposited into the fireball ($\sim 10^{52}$ ergs during the collapse to a black hole of $10^6\ M_\odot$), the distortion produced in the cosmic background radiation through the scattering of hot electrons (Sunyaev-Zeldovich effect) occurs on a very small angular scale ($\theta \lesssim 10^{-10}$ arcsec) and is therefore undetectable.

In the FS model a potential problem, as a GRB source, is related to the "baryon-loading,"[2] that is, the confinement of the electron/positron/photon fireball by the baryons which could carry energy of it in the form of kinetic energy, thus diminishing the amount of energetic photons (the gamma ones). This suggests that the region at several Schwarzschild radii from the supermassive star core should have extremely low baryon density. There are at least two ways to avoid the excessive baryon loading: rotation of the star producing the flattened collapse or the collapse of a dense star cluster instead of a single object. This could result in a different event rate for the GRBs and the GW bursts, not all GW bursts being related to GRBs in the present scenario since the baryons could block the γ-rays.

Even if the GRBs and GW bursts have completely different event rates, either because the source of GWs does not

[2] There are many papers in the literature discussing aspects related to the injection of energy (including the baryon-loading problem) associated with GRBs. In particular, we refer the reader for the papers of Shemi & Piran 1990; Kobayashi, Piran, & Sari 1999, and Fuller, Pruet, & Abazajian 2000.

produce a GRB at all or because the gamma radiation is blocked, it would be possible to verify the FS scenario by looking for GRBs once GW bursts associated with SMBH formation are observed and identified. There will be a time interval between the GRB and the GW burst because the types of radiation are generated in different ways. The generation of the GRB depends on a series of physical processes after the collapse of the core, e.g., the generation of the fireball to accelerate the matter to the ultrarelativistic regime when the kinetic energy in the fireball could be converted to γ-rays. The GWs, on the other hand, are mainly produced when the SMBH is formed, through the excitation of its quasi-normal modes. A detailed modeling is required however to evaluate the time interval between the GRB and the GW burst.

Using the *LISA* observatory to detect GW bursts related to the SMBHs formation, one could find their GW amplitudes, the characteristic frequencies and also the formation rate of SMBHs. If we also find the redshift associated with the events (by observing in the electromagnetic window) we will be able to obtain the SMBH masses and the GW efficiency using the model proposed here. By comparing the SMBH formation GW event rates with the GRB rates, one could also infer what fraction of an ensemble of SMBHs had conditions to generate GRBs and to impose constraints on the FS scenario. Then in the present study we are proposing an independent way to check FS model through GW astronomy.

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