# Superluminal and slow light propagation in SOA based on coherent population oscillations

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Based on the general mechanism of the coherent population oscillations, we propose the fundamental harmonic fractional delay (FHFD) to evaluate the superluminal and slow light propagation in semiconductor optical amplifier (SOA). The sinusoidal and square-wave signals in SOA are investigated with the propagation equations. It is shown that the superluminal and slow light always accompany the signal distortion, and FHFD depends on the signal distortion as well as the incident power, the modulation frequency, and the optical gain.

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The effect of slow light has attracted increasing attention because it can be used in all-optical signal processing, all-optical buffer, optical signal delay, and so on[1,2]. Bigelow *et al.*[3–5] researched the sinusoidal signal modulated on the wavelength of 514.5 nm propagating in ruby crystal, which shows the slow light effect. The group velocity is reduced to be 57.5 m/s. And then they reported the superluminal and slow light in an alexandrite crystal at the room temperature, the group velocities as slow as 91–800 m/s were measured. However, the time delay measurement in the experiment has the problem with choosing the reference points, the time delay or phase delay will be different at different particular reference points of the probe and reference signals. Based on the coherent population oscillations (CPOs) theory, Gehring *et al.*[6] reported the observation of backward pulse propagation through a medium with a negative group velocity. They analyzed that the peak of the pulse does propagate backward inside the fiber.

Moon *et al.*[7] reported slow light effect in the gain regime with higher power than that of fast light, utilizing the anomalous gain characteristic in a gain-clamped semiconductor optical amplifier (SOA). They reported that the phase delay is 44° when the signal is 5 GHz sinusoidal signal, and the phase delay is 34° for 10 GHz sinusoidal signal. Kondratko *et al.*[8] reported that they achieved a phase of 160° and a scaling factor of 4 at 1 GHz using the cascade of four quantum well SOAs. Thus, the reports are all the slow light of sinusoidal waveform signal with the particular reference points.

However, the time delay or phase delay is measured by the time difference between both particular reference points of both the probe and the reference signals without the consideration of the signal distortion for sinusoidal signal or single Gaussian signal. It is obvious that this method is not popular or effective to evaluate the behavior for different waveform signals[9,10]. Derived from the general mechanism of CPO, the superluminal and slow light mainly result from the interaction between the modulated incident signal as pump and its first sideband as probe. So we can only consider the phase delay of the fundamental harmonic between the pump and probe light to evaluate the superluminal or slow light without consideration of any higher order harmonics. In this letter, we propose the fundamental harmonic fractional delay (FHFD) to evaluate the superluminal and slow light propagation in SOA. The sinusoidal and square-wave waveform signals in SOA are investigated with the dependences on the input power as well as the modulation frequency and the pump power.

The group index in absorption or gain medium are given by[3–5,11,12]

$$n_{\mathrm{a}} = n(\delta) - \frac{\alpha_0 c T_1}{2}\frac{I_0}{1+I_0}\left(\frac{1}{(2\pi T_1\delta)^2 + (1+I_0)^2}\right),$$

$$n_{\mathrm{g}} = n(\delta) - \frac{g_0 c T_1}{2}\frac{I_0}{1+I_0}\left(\frac{1}{(2\pi T_1\delta)^2 + (1+I_0)^2}\right), \qquad (1)$$

where $g_0$ and $\alpha_0$ are the unsaturated values of the gain and absorption coefficients; $\delta(\omega_p - \omega_s)/2\pi$ is the pump–probe beat frequency; $I_0 = I/I_{\mathrm{sat}} = \Omega^2 T_1 T_2$ is the light intensity $I$ normalized to the saturation intensity $I_{\mathrm{sat}}$ of SOA;. $T_1$ is the relaxation time of the population inversion of the SOA; $T_2$ is the dipole moment dephasing time; $\Omega = 2\mu E/\mathrm{H}$ is the Rabi frequency; $\mu$ is the dipole matrix elements; $E$ is the amplitude of the incident field $E_{\mathrm{in}}$ without injection current, or $E = \sqrt{G}E_{\mathrm{in}}$ with injection current and hence $G$ is the optical gain through the SOA.

The time delay in SOA is given by $\tau_\mathrm{d}=L(n_\mathrm{g}(\delta)-n_\mathrm{g0})/c$. So the fractional delay is

$$F=\tau_\mathrm{d}/T \approx L\delta n_g(\delta)/c. \tag{2}$$

Figure 1(a) shows the fractional delay (positive means slow light) at different modulation frequencies for different incident powers (i.e., the signal power) and its inset shows that the fractional delay increases with the incident power Fig. 1(b) shows the fractional delay (negative means superluminal light) at different modulation frequencies for different gains (according to different injection currents) and its inset shows that the fractional delay increases with the gain. There is a peak value of the fractional delay at a particular frequency for every case, which is limited by the carrier lifetime of the SOA, which is discussed in detail below.

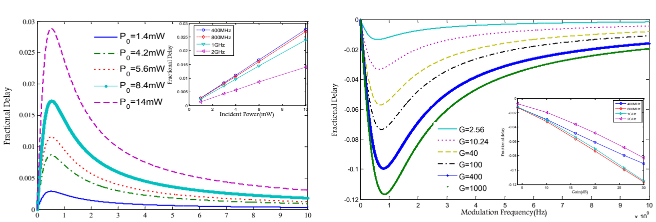
Evaluating the superluminal and slow light with the fractional delay defined as Eq. (2) is reasonable, but there are some problems in the experiment measurements. The time delay is a relative time difference between two time points, and we found that choosing different particular points result in different time delay or phase delay due to the distortion of the probe signal in our previous works[9,10]. From the general mechanism of CPO, the superluminal and slow light mainly result from the interaction between the modulated incident signal as pump and its first sideband as probe. So we can only consider the phase delay of the fundamental harmonic between the pump and probe light to evaluate the superluminal or slow light without consideration of any higher order harmonics. This evaluation can eliminate the dependence on the reference point for any wave pattern[9,10].

The Fourier transformation of the incident signal $P_\mathrm{s}(0,t)$can be written as

$$P_\mathrm{ref}=\int_{-\infty}^{\infty}P_\mathrm{s}(0,t)e^{-j2\pi ft}\mathrm{d}t=P_0+P_1e^{j\varphi^1_\mathrm{ref}(f_m)}+\sum_{n=2}^{\infty}P_n e^{j\varphi^n_\mathrm{ref}(nf_m)}, \tag{3}$$

where $\varphi^n{}_\mathrm{ref}(nf_m)$ is the reference signal's phase of the $n$th-order harmonic and $P_n$ is the amplitude of the $n$th-order harmonic.

The Fourier transformation of the probe signal can be written as




(a) (b)

Fig. 1. Fractional delay with a sinusoidal modulated signal injected into SOA, while the incident powers are 1.4, 4.2, 5.6, 8.4, and 14 mW (a)without injection and (b) with different injection currents (different gain) for a particular incident power of 1 mW. The insets show FHFD of signal at different modulation frequencies 400 and 800 MHz, 1 and 2 GHz with different incident powers.

$$P_\mathrm{probe}=\int_{-\infty}^{\infty}P_\mathrm{s}(L,t)e^{-j2\pi ft}\mathrm{d}t=u_0+u_1e^{j\varphi^1_\mathrm{probe}(f_m)}+\sum_{n=2}^{\infty}u_n e^{j\varphi^n_\mathrm{probe}(nf_m)}, \tag{4}$$

where $u_0$ is the DC component, $u_\mathrm{n}$ and $\varphi^n{}_\mathrm{probe}(nf_\mathrm{m})$ are the amplitude and the phase of the $n$th harmonic component. In fact, the time difference between any order harmonics of both the probe and reference signals can be used to discuss the superluminal or slow light effects in the active medium. For the fundamental harmonics, $N=1$, $F_1$ is greater than any other $F_N$, so $F_1$, so-called FHFD, is always used to evaluate the superluminal and slow light effect in active media. Obviously for the fundamental, the time delay for all points of these light beams is the same, so FHFD can be used to evaluate the superluminal or slow light defined by

$$\mathrm{FHFD}=[\varphi^1_\mathrm{probe}(f_\mathrm{m})-\varphi^1_\mathrm{ref}(f_\mathrm{m})]/2\pi, \tag{5}$$

and total harmonic distortion (THD) factor of the probe signal can be defined to evaluate the distortion of the signal transmission through the SOA as

$$\mathrm{THD}=\sqrt{u_2^2+u_3^2+\cdots+u_n^2}/u_1. \tag{6}$$

A periodic signal with arbitrary waveform input into the SOA can be expressed as

$$P_\mathrm{s}(0,t)=P_0\left[1+m\Psi(\delta t)\right], \tag{7}$$

where $\Psi(\delta t)$ is the periodic signal, $\delta$ is the modulation frequency; $P_0$ is the average power; and $0<m<1$ is the modulation depth. The propagation equations in SOA can be given by[13]

$$\begin{aligned}\frac{\partial A(z,T)}{\partial z}&=\frac{i}{2}\beta_2\frac{\partial^2A(z,T)}{\partial T^2}+\frac{1}{2}\frac{g-\varepsilon_2P_\mathrm{s}^2}{1+\varepsilon_1P_\mathrm{s}}A(z,T)\\&-\frac{i}{2}\left[a_\mathrm{N}g-a_\mathrm{T}\frac{\varepsilon_1gP_\mathrm{s}+\varepsilon_2P_\mathrm{s}^2}{1+\varepsilon_1P_\mathrm{s}}A(z,T)\right]\\&-\left(\Gamma_2\gamma+i\Gamma'_2\frac{\omega_0}{c}n_2\right)\frac{1}{\sigma}P_\mathrm{s}A(z,T)-\frac{1}{2}a_\mathrm{int}A(z,T),\end{aligned} \tag{8}$$

$$\frac{\partial g}{\partial T}=\frac{g_0-g}{T_1}-\frac{1}{E_\mathrm{sat}}\frac{g-\varepsilon_2P_\mathrm{s}^2}{1+\varepsilon_1P_\mathrm{s}}P_\mathrm{s}-\frac{\Gamma_2a_N\gamma}{h\nu\sigma^2}P_\mathrm{s}^2, \tag{9}$$

where $A(z,\ T)$ is the envelope of the incident field; $P_\mathrm{s}=|A(z,\ T)|^2$ is the incident signal power; $T_1$ is the relaxation time of the population inversion of the SOA; $\omega_0$ is the central angular frequency. Solving Eqs. (7)–(9), we can get the probe signal output from the SOA. Applying Fourier transformation to the incident signal and the output probe signal, then the THD and the FHFD can be obtained from Eqs. (5) and (6) to evaluate the slow light and superluminal light in the SOA. The simulation parameters are listed in Table 1[13].

The sinusoidal and rectangle waves are injected into the SOA, respectively, to check the power dependence, while the incident average powers range from 1 to 14 mW

**Table 1.** List of Parameters of SOA Simulation

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $L$ (m) | $5\times10^{-4}$ | $a_N$ (m$^2$) | $3\times10^{-20}$ |
| $\alpha_{\text{int}}$ (m$^{-1}$) | $2\times10^3$ | $\gamma$ (m/W) | $3.7\times10^{-10}$ |
| $\alpha_N$ | 5.0 | $\alpha_T$ | 1.1 |
| $\varepsilon_1$ (W$^{-1}$) | 0.2 | $\varepsilon_2$ (W$^{-2}$) | 200 |
| $\Gamma_2$ | 0.5 | $\Gamma'_2$ | 0.4 |
| $g_0$ (dB) | 30 | $n_2$ (m$^2$/W) | $-3.5\times10^{-16}$ |
| $\beta_2$ (s$^2$/m) | $5\times10^{-24}$ | $E_{\text{sat}}$ (J) | $6.91\times10^{-12}$ |
| $\sigma$ (m$^2$) | $1.5\times10^{-12}$ | $T_1$ (s) | $3\times10^{-10}$ |

with the optical gain $G = 0.01$, and the THD and FHFD are shown in Fig. 2. Figure 2(a) shows the THD of sinusoidal signal, which increases with increasing incident power, and decreases with the modulation frequency as shown in its inset. Figure 2(c) shows the THD of rectangle signal, which is contrary to the behavior found for sinusoidal signal, which decreases with increasing incident power, and there is a valley value at a particular frequency. Figures 2(b) and (d) show that the FHFD of both sinusoidal and rectangle waveform signals increase with the incident powers, and there is a peak value at a particular modulation frequency.

The sinusoidal and rectangle waves with the average power of 1 mW are injected into the SOA, respectively, to check the dependence on the optical gain (equivalent to injection current), while the optical gain $G$ = 2.56, 10.24, 40, 100, 400, 1000. Owing to the gain, the signal propagation in SOA will be superluminal light. Figures 3(a) and (c) show the THD of sinusoidal and rectangle signals, all increase with the increment of the gain, and there is a peak value at a particular modulation frequency. Figures 3(b) and (d) show the FHFD of these signals,

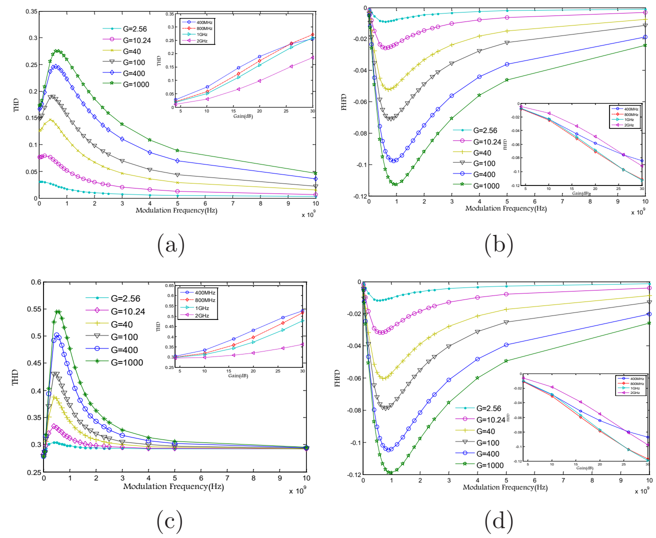



Fig. 3. THD and FHFD as a function of the optical gain in the SOA for the sinusoidal and rectangle waveform signals: (a) THD of sinusoidal signal, (b) FHFD of sinusoidal signal, (c) THD of rectangle signal, and (d) FHFD of rectangle signal.

and the negative value means the superluminal light effect. The value of the FHFD increases with the increment of the gain, and there is a negative peak value at a particular modulation frequency.

From the above simulations, there is a peak value at a particular modulation frequency no matter for superluminal or slow light. Figure 4(a) demonstrates the maximum fractional delay for slow light obtained from the CPO theory and the FHFD for sinusoidal and rectangle waveform signals at different incident optical powers. It is shown that the maximum fractional delay linearly increases with the incident power, and the corresponding modulation frequency also increases with the incident power as shown in Fig. 4(b). The rectangle

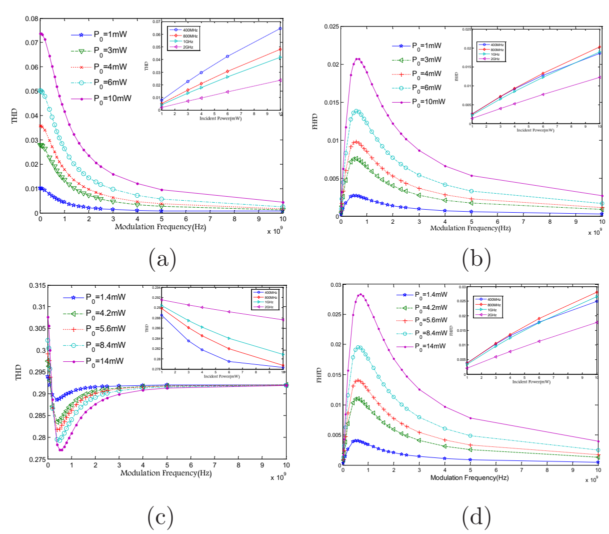



Fig. 2. THD and FHFD as a function of the modulation frequency and the incident power: (a) THD of sinusoidal signal, (b) FHFD of sinusoidal signal, (c) THD of rectangle signal, and (d) FHFD of rectangle signal.

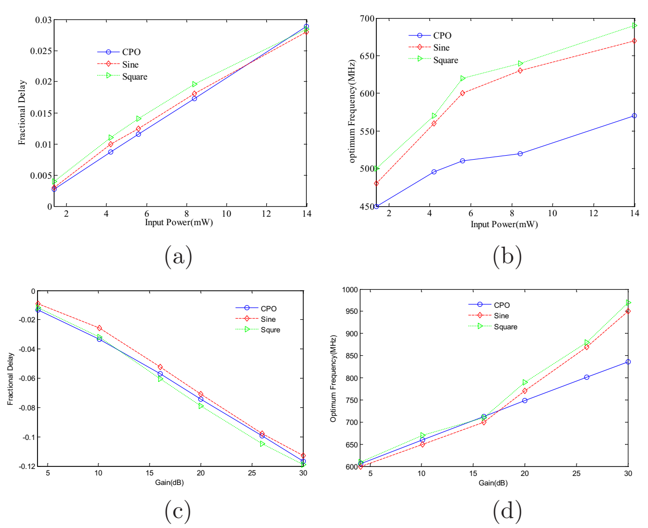


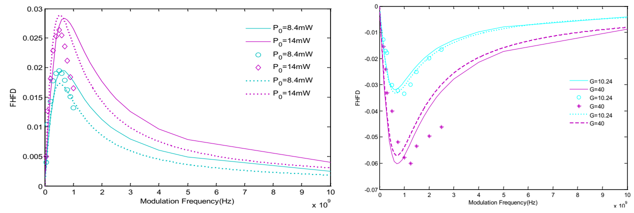

Fig. 4. Dependence of the maximum fractional delay and its corresponding modulation frequency: (a) and (b) show the dependences on the incident power; (c) and (d) show the dependences on the gain.

signal's maximum fractional delay is greater than the CPO's, the optimum frequency from CPO is smaller than sinusoidal and rectangle signals, which is assumed to be contributed from the signal distortion.

Figure 4(c) shows the maximum fractional delay for superluminal light obtained from the CPO theory and the FHFD of sinusoidal and rectangle signals. It is clear that the negative maximum fractional delay also linearly increases with the increment of the gain and its corresponding modulation frequency increases too (Fig. 4(d)). The maximum fractional delay and their corresponding modulation frequencies of CPO, sinusoidal, and rectangle signals have good agreement.

The experimental setup is shown in Fig. 5, part 1 is the source, part 2 is the superluminal/slow light unit, and part 3 is the detection. In part 1, the light output from a wavelength tunable laser is modulated by an $LiNbO_3$ modulator (LN-MOD), which is driven by a function generator (FG) with signals of variable waveform, modulation frequency, and modulation depth. The modulated light is amplified by a gain clamped EDFA, which is a ring structure including two wavelength-division multiplexers (WDMs), two isolators (ISOs), a variable optical attenuator (VOA), and a conventional EDFA. Then the amplified modulated light with enough power enters part 2, the superluminal/slow light unit, where a 98:2 coupler CP1 is used to split the light into two beams as the probe and reference. The probe passes through the SOA, which is the active media for the contribution to the CPOSs for slow light or fast light, and then enters the detection part 3 with one half of the reference (the other half is monitored by a power meter (PM)), which has passed through a section of SMF to compensate the optical path difference between the probe and the reference. From the time domain data obtained from the oscilloscope (OSC), the THD and FHFD can be calculated by employing the Fourier transformation.

Figure 6 shows an FHFD experiment as the scattered dots for rectangle signal with the incident power of 8.4 and 14 mW or fixed incident power of 1 mW with optical gains of 10.24 and 40, superpositioning with the results from CPO theory and simulations. It shows good agreement among the CPO, the simulations, and the measurements.




(a) (b)

Fig. 6. FHFD experiment (scattered open dots) of the rectangle signal with the incident powers of (a) 8.4 and 14 mW or fixed incident power of 1 mW with optical gains of (b) 10.24 and 40, with the CPO theory (dotted lines) and simulations (solid lines).

When the slow or superluminal light effects are employed for optical telecommunications, the signal is usually square pulse series, and the signal distortion should be as weak as possible to obtain enough FHFD. In order to evaluate trade-off of the THD and FHFD for telecommunications, a parameter $Q$ is simply defined as the quotient between the FHFD and THD as $Q$ = FHFD/THD, positive $Q$ means slow light and negative $Q$ means superluminal light. To check the $Q$, a rectangle wave with different average incident powers and different optical gains is injected into the SOA (Figs. 2 and 3). From Fig. 7, we can find the best working points with smaller THD and bigger FHFD. Figure 7(a) demonstrates the $Q$-dependence on incident power and the modulation frequency and Fig. 7(b) shows the $Q$-dependence on optical gain and modulation frequency.

In conclusion, based on the general mechanism of the CPO, we propose the FHFD to evaluate the superluminal and slow light effect on SOA. With some simulations for the sinusoidal and square-wave signals in SOA based on the propagation equations, it is shown that the superluminal and slow light always accompany the signal distortion, which is evaluated by the THD, and the FHFD depends on the incident power as well as the modulation frequency and the pump power. For both superluminal and slow light effect, there are peak values of the FHFD at a particular modulation frequency, which is limited by the lifetime of the carrier. The FHFD increases with the input optical power and the optical gain due to the injection current, and the

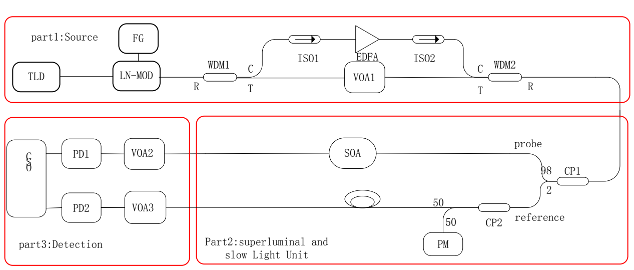
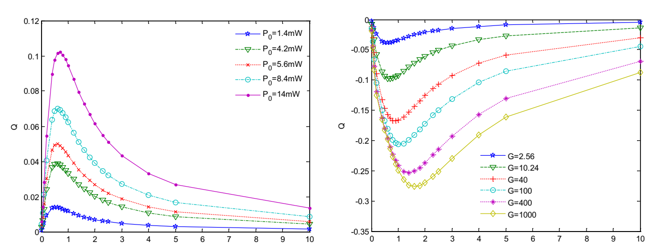



Fig. 5. Experimental setup to check the effect of superluminal and slow light in SOA. TLD, tunable laser diode; PD, photo-detector.




(a) (b)

Fig. 7. Calculated $Q$ parameters for SOA. $Q$-dependence on (a) incident power and (b) optical gain.

peak value and its corresponding modulation frequency also increase with the incident power and the gain.

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