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RESEARCH ARTICLE

Optimizing Replenishment Base on Order Structure in Combined Automatic Warehouse System

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ABSTRACT This paper presents a new combined automatic warehouse system that can not only realize automatic storage and sorting processes, but also process automatic replenishment requests according to inventory demand. In this system, there are two storage areas: one for the pallets and the other for the totes. The pallet storage area can process the storage and picking of pallets, whereas the tote storage area can handle case storage and picking. In addition, this system automatically initiates a replenishment request from the pallet storage area to the tote storage area when the storage area is lower than the safety stock. This study uses the characteristics of the order matrix to study the influence of order structure on replenishment with four evaluation parameters: the workload of two storage areas, the number of used storage positions in the tote storage area, and replenishment time. The numerical analysis part sets different order parameter values, including the order density, order strength, and wave size, to analyze the impact of multiple order structures on the system performance, which helps warehouse operation managers decide the replenishment strategy parameters.

INDEX TERMS Automatic warehouse system, replenishment, order structure.

I. INTRODUCTION

The combined automatic warehouse system (CAWS) is a new variation of the automated storage and retrieval system (AS/RS) and the shuttle-based storage and retrieval system (SBS/RS). CAWS is divided into two storage areas depending on the storage container, which are defined as the crane-based storage and retrieval system (CBS/RS) and the shuttle-base storage and retrieval system (SBS/RS). As shown in Fig. 1, CBS/RS is used to store and pick the pallets. The green module in the middle of the aisle is the crane, which runs in each aisle and provides pallet transportation in both the horizontal and vertical directions. The blue module between the I/O point and the I/O workstation provides horizontal transportation of the pallets. The other subsystem is SBS/RS, which stores totes and uses shuttles and lifters to transport totes. In the SBS/RS system, each aisle has a shuttle car at each tier, as shown by the orange module in Fig 1. The shuttle provides horizontal transport of totes, whereas the lifter provides vertical transport of totes. The purple module located at the beginning of the aisle is the lifter dedicated to serving the exclusive aisle. Therefore, CAWS has the basic characteristics of AS/RS, such as efficient space utilization and large storage capacity. Moreover, CAWS also has the same flexibility and high throughput as SBS/RS. Consequently, as the number of orders becomes increasingly fragmented, CAWS is a better solution in scenarios with greater storage density requirements and high throughput.

Most distribution centers operate order picking on a wave release basis. In CAWS, the pallets unloaded from the input truck are received and stored in the CBS/RS. When the order arrives, the system first checks if the memory reserve of the SBS/RS is sufficient. If the stock is sufficient, execute the order-picking task immediately. If the stock is insufficient,

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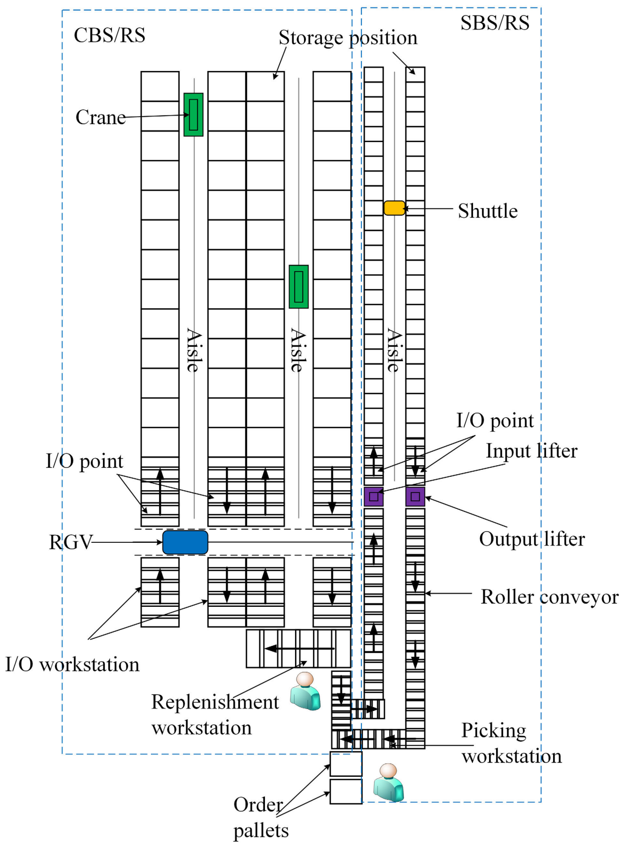


FIGURE 1. The top view description of the CAWS.

initiate the automatic replenishment task from CBS/RS to SBS/RS first. In this replenishment operation, the pallets stored in the CBS/RS are conveyed to the replenishment workstation and then detached. The SBS/RS supplies empty totes to store the cases detached from the pallet and then stores these totes for cases and item picking in this wave. To reduce the system workload of replenishment, orders need to be analyzed and consolidated; that is, the stock keeping units (SKUs) of all orders in one wave need to be consolidated to concentrate replenishment. Therefore, it is important to analyze the influence of the order structure (such as the size of the order wave, the density of the order wave, and the intensity of the order wave) on the replenishment efficiency.

II. RELEVANT WORKS

There are few studies on CAWS; therefore, this paper reviews the studies on AS/RS and SBS/RS, which can also be used as an analysis of the research status of the combined system of CAWS. Azadeh et al. [1] provided a comprehensive overview of robotic and automated warehouse systems, including crane-based storage and retrieval systems, shuttle-based compact storage and retrieval systems, and robotic mobile fulfilment systems. Their study provided modeling solutions related to system analysis, design optimization, and operational planning. Hausman et al. [2] published research articles on AS/RS in 1976. They studied storage assignment and optimized it by defining the class-based turnover rate. Since then, research on AS/RS has gained momentum and hundreds of papers have been published. Research on AS/RS mainly focuses on two aspects: system performance, such as system throughput, order cycle time, and system storage space, and operational policy, such as task scheduling and storage policy [3], [4], [5], [6].

In recent years, one automated warehouse system that uses shuttles and lifters to transport the totes has been named vehicle-based storage and retrieval system (AVS/RS), which is very popular in practice and research with high flexibility and throughput. Malmborg [7], [8], [9] provided an optimization model to study the storage and retrieval cycle time for tier-to-tier AVS/RS, which can calculate the system utilization and throughput capacity. Some researchers have used mathematical methods to calculate the performance parameters of the system, such as Fukunari and Malmborg [10], Roy et al. [11], [12] and Ekren et al. [13], [14], who used queuing theory to estimate resource utilization and system throughput. In addition, there is another system called tier-captive AVS/RS, which is receiving considerable attention from researchers. This system has dedicated vehicles, which cannot be transferred to another tier and can only service one tier. Marchet et al. [15], Wang et al. [16], Zou et al. [17], and Tappia et al. [18] proposed an applicable queuing network to estimate the performance of a system and compare operation strategies. Yang [19] analyzed the single-task operation time. The performance of the shuttle warehouse system was evaluated and the operational flow of the system was introduced.

We found few articles in system studies that considered the effect of the order structure. Different order structures are suitable for different automatic picking systems and operation strategies. Shen [20] provided a grid method to define the order structure and studied the system performance by comparing sequential and simultaneous zoning. Wang [21] studied the AS/RS and Carousel systems and provided an applicability analysis with different-order structures. Ma [24] focused on regression analysis to calculate the quantity of replenishment based on the order structure. Ma [25] provided a strategy for optimizing the output part of the system considering the order structure. Through the proper splitting of orders, the output time is shortened, and the efficiency of the system is improved. Liu [26] set labels for orders: Dynamic Order-Based and Dynamic Order-Based with Threshold in ARS systems are studied, which is similar to how we mark SKUs in orders.

The remainder of this paper is organized as follows. We first describe the replenishment transaction in CAWS, propose the main assumptions and notations, and analyze the throughput of CBS/RS and SBS/RS. Subsequently, an order matrix was established to study the order structure. The replenishment strategy is described and modelled. Finally, scenarios with multiple order structures were designed to analyze the influence of the order structure on replenishment.

III. SYSTEM DESCRIPTION

A. REPLENISHMENT TRANSACTION

In the replenishment transaction in the CAWS, the cases stored in the CBS/RS are replenished with the SBS/RS. The flow of replenishment transactions in the CAWS is shown in Fig 2. When the order arrives, the warehouse computer first analyzes the order according to the three types of information and calculates the replenishment task.

- SKU information, including the types and sizes of SKU. The type of SKU can be used as a reference to determine the appropriate replenishment method, and the size of SKU can be used to calculate the number of empty bags required.;
- Stock information, including the number of occupied and unoccupied storage positions, can be used to calculate replenishment quantity.
- Order information, which determines the quantity to be replenished.

At the beginning of the replenishment transaction, the warehouse computer first detects whether there are sufficient empty totes in the replenishment workstation. If there are not enough empty totes, the SBS/RS processes the empty tote retrieval transaction (see Appendix). If there are sufficient empty totes, this process does not need to be executed. The inventory for replenishment transactions is provided by the CBS/RS, which performs the pallet retrieval process (see Appendix). When both the pallets and empty totes reach the replenishment workstation, the picker moves the cases from the pallets to the empty totes. SBS/RS then processes the storage transaction to store the tote (see the Appendix). If any cases remain in the pallet, the CBS/RS should handle the storage transaction to return the pallet (see Appendix).

B. MAIN ASSUMPTION

The assumption of CAWS in this study are listed below:

- As in most previous studies (e.g. Ekren et al. [14]; Lerher et al. [21]), the system operates with a random storage and retrieval policy. Under this policy, the retrieval and storage transactions can be assigned to any storage position with the same probability.
- We assume that the equipment in CAWS operates in dual-command cycles, that is, a storage transaction and retrieval transaction in each cycle.
- The dwell-point policy of cranes, RGVs, shuttles, and lifters follows the point-of-service-completion (POSC).
- The transporters manage the transaction queue in the first-come, first-served (FCFS) discipline.
- One storage position contained one pallet or tote. Each pallet or tote can contain many cases, depending on the SKU package size, but only one SKU type.
- We assume that there is enough stock in CBS/RS to satisfy replenishment, that is, there is no replenishment to CBS/RS from outside.

C. MAIN NOTATION

The main notation of this study are as follows:

$N_{C,A}, N_{C,T}, N_{C,C}$:	Number of CBS/RS storage aisles, tiers and columns
$N_{S,A}, N_{S,T}, N_{S,C}$:	Number of SBS/RS storage aisles, tiers and columns
$u_{S,w}, u_{S,d}, u_{S,h}$:	Unit gross width, depth and height per storage position of SBS/RS
$u_{C,w}, u_{C,d}, u_{C,h}$:	Unit gross width, depth and height per storage position of CBS/RS
$N_{C,R}, N_{C,I,P}, N_{C,I,W}$:	Number of RGVs, I/O points and I/O workstations in the CBS/RS
vv_{cr}, va_{cr} :	Vertical maximum velocity and acceleration/ deceleration of crane
hv_{cr}, ha_{cr} :	Horizontal maximum velocity and acceleration/deceleration of crane
v_R, a_R :	Maximum velocity and acceleration/ deceleration rate of RGV
v_l, a_l :	Maximum velocity and acceleration/ deceleration rate of the lifter
v_s, a_s :	Maximum velocity and acceleration/ deceleration rate of the shuttle
t_{cr}, t_l, t_s, t_R :	Fixed time required for the crane, lift, shuttle and RGV to load or unload the tote
T_{cr}, T_R, T_l, T_s :	Mean service time of crane, RGV, lift and shuttle
Th_C, Th_S :	Throughput of CBS/RS and SBS/RS
Q_p, Q_t :	Quantity of full pallet and tote (cases)
ρ	Proportion of high turnover SKU
$Q_{r,j}$	Quantity of jth SKU replenishment
D_j	Workload of jth SKU replenishment

D. MODELLING THE CAWS

1) ANALYZE THROUGHPUT OF CBS/RS

The number of pallets retrieved per unit time is defined as the throughput of the CBS/RS. As in Marchet et al. [15], the throughput is defined by the bottleneck of the system (i.e., the throughput of the system equals the throughput of bottlenecks), which is related to the rack configuration and mechanical properties.

Throughput of CBS/RS is calculated as follows:

$$TH_C = \min\left\{\frac{N_{C,A}}{T_{cr}}, \frac{N_{C,R}}{T_R}\right\} \tag{1}$$

where T_{cr} and T_R represent the average cycle times of each crane and RGV, $\frac{N_{C,A}}{T_{cr}}$ and $\frac{N_{C,R}}{T_R}$ represents the throughput of all cranes and RGVs.

The average dual-command cycle time of cranes is divided into seven steps:

1) pick up the pallet ai I/O point;
2) travel from I/O point to the storage destination position;
3) release the pallet;
4) Move to the retrieval destination position;
5) pick up the pallet;
6) Return to the I/O point;
7) Release the pallet.

The T_{cr} can be calculated using the method provided by Ma [24]. T_{cr} can be calculated using equation (2), according to the assumption of a random storage and retrieval policy.

$$\begin{aligned} T_{cr} &= \frac{2}{N_{C,T} \times N_{C,C}} \times \sum_{i=1}^{N_{C,T} \times N_{C,C}} T_{cr}^{i}(i) \\ &\quad + \frac{1}{\left(N_{C,T} \times N_{C,C}\right)^2 - \left(N_{C,T} \times N_{C,C}\right)} \\ &\quad \times \sum_{i=1}^{N_{C,T} \times N_{C,C}} \sum_{j=1}^{N_{C,T} \times N_{C,C}} T_{cr}^{ii}(i,j) + 4 \times t_{cr} \\ &= \frac{2}{N_{C,T} \times N_{C,C}} \\ &\quad \times \sum_{i=j}^{N_{C,T} \times N_{C,C}} \max\left\{T_{cr}^{i}(i), T_{cr}^{i}{}_{h}(i)\right\} \\ &\quad + \frac{1}{\left(N_{C,T} \times N_{C,C}\right)^2 - \left(N_{C,T} \times N_{C,C}\right)} \\ &\quad \times \sum_{i=1}^{N_{C,T} \times N_{C,C}} \sum_{\substack{j=1 \\ i \neq j}}^{N_{C,T} \times N_{C,C}} \max\left\{T_{crv}^{ii}(i,j), T_{cr}\ddot{i}, (i,j)\right\} \\ &\quad + 4 \times t_{cr} \end{aligned} \tag{2}$$

where $T_{cr}^{\mathrm{iv}}(i)$ and $T_{cr}^{ih}(i)$ represent the travel times in the vertical and horizontal directions from the I/O point to the destination storage position, respectively. $T_{cr}^{\mathrm{iiv}}(i,j)$ and $T_{cr}^{\mathrm{iih}}(i,j)$ represent the travel time in the vertical and horizontal directions from a storage position to the destination retrieval position. The travel time was divided into two formulations depending on whether the peak velocity was reached and was calculated as follows:

$$\begin{aligned} &T_{cr_v}^{i}(i) \\ &= \begin{cases} 2 \times \dfrac{vv_{cr}}{va_{cr}} + \dfrac{D_v(i) - \frac{vv_{cr2}}{va_{cr}}}{vv_{cr}}, & D_v(i) > \dfrac{vv_{cr}{}^2}{va_{cr}} \\ 2 \times \sqrt{\dfrac{D_v(i)}{va_{cr}}}, & D_v(i) \leq \dfrac{vv_{cr}^2}{va_{cr}} \end{cases} \end{aligned} \tag{3}$$

$$\begin{aligned} &T_{cr_v}^{i}(i) \\ &= \begin{cases} 2 \times \dfrac{vv_{cr}}{va_{cr}} + \dfrac{D_v(i) - \frac{vv_{cr2}}{vcr}}{vv_{cr}}, & D_v(i) > \dfrac{vv_{cr}^2}{va_{cr}} \\ 2 \times \sqrt{\dfrac{D_v(i)}{va_{cr}}}, & D_v(i) \leq \dfrac{vv_{cr2}}{va_{cr}} \end{cases} \end{aligned} \tag{4}$$

where $D_v(i)$ and $D_h(i)$ represent the vertical and horizontal travel distance of cranes:

$$D_v(i) = \left\lceil \frac{i-1}{N_{C,C}} \right\rceil \times u_{C,h} \tag{5}$$

$$D_h(i) = \left(i - \left\lfloor \frac{i}{N_{C,C}} \right\rfloor \times N_{C,C}\right) \times u_{C,w} \tag{6}$$

$T_{cr}^{\mathrm{iiv}}(i,j)$ and $T_{cr}^{\mathrm{iih}}(i,j)$ can be calculated by:

$$\begin{aligned} &T_{cr}^{ii}(i,j) \\ &= \begin{cases} 2 \times \dfrac{vv_{cr}}{va_{cr}} + \dfrac{D_v(i,j) - \frac{vv_{cr2}}{va_{cr}}}{vv_{cr}}, & D_v(i,j) > \dfrac{vv_{cr2}}{va_{cr}} \\ 2 \times \sqrt{\dfrac{D_v(i,j)}{va_{cr}}}, & D_v(i,j) \leq \dfrac{vv_{cr}}{va_{cr}} \end{cases} \end{aligned} \tag{7}$$

$$\begin{aligned} &T_{cr_h}^{ij}(i,j) \\ &= \begin{cases} 2 \times \dfrac{hv_{cr}}{ha_{cr}} + \dfrac{D_h(i,j) - \frac{hv_{cr}{}^2}{ha_{cr}}}{hv_{cr}}, & D_h(i,j) > \dfrac{hv_{cr}{}^2}{ha_{cr}} \\ 2 \times \sqrt{\dfrac{D_h(i,j)}{ha_{cr}}}, & D_h(i,j) \leq \dfrac{hv_{cr}{}^2}{ha_{cr}} \end{cases} \end{aligned} \tag{8}$$

where $D_v(i,j)$ and $D_h(i,j)$ represent the vertical and horizontal travel distance of cranes:

$$D_v(i,j) = \left\lceil \frac{|i-j|}{N_{C,C}} \right\rceil \times u_{C,h} \tag{9}$$

$$\begin{aligned} D_h(i,j) = \Bigg| &\left(i - \left\lfloor \frac{i}{N_{C,C}} \right\rfloor \times N_{C,C}\right) \\ &- \left(j - \left\lfloor \frac{j}{N_{C,C}} \right\rfloor \times N_{C,C}\right) \Bigg| \times u_{C,w} \end{aligned} \tag{10}$$

The dual-command cycle of the RGV is divided into $\frac{1}{2} \times N_{C,I,P} \times \frac{1}{2} \times N_{C,I,W}$ scenarios. The average cycle time T_R was calculated as follows:

$$\begin{aligned} T_R = &\frac{1}{\frac{1}{2} \times N_{C,I,P} \times \frac{1}{2} \times N_{C,I,W}} \\ &\times \sum_{i=1}^{\frac{1}{2} \times N_{C,I,P} \times \frac{1}{2} \times N_{C,I,W}} T_R(i) + 2 \times t_R \end{aligned} \tag{11}$$

where $T_R(i)$ represents the travel time in each scenario.

$$T_R(i) = \begin{cases} 2 \times \frac{v_R}{a_R} + \frac{D(i) - \frac{v_R^2}{a_R}}{v_R}, D(i) > \frac{v_R^2}{a_R} \\ 2 \times \sqrt{\frac{D(i)}{a_R}}, D(i) \leq \frac{v_R^2}{a_R} \end{cases} \tag{12}$$

2) ANALYZE THROUGHPUT OF SBS/RS

As in the CBS/RS case, the throughput of the SBS/RS is a function of bottlenecks owing to the use of different resources (i.e., shuttles and lifters), which represents the number of tokens that the system can retrieve per time unit.

Throughput of SBS/RS is calculated as follows:

$$Th_S = min\frac{N_{S,T}}{T_s}, \frac{N_{S,A}}{T_l} \tag{13}$$

where T_s and T_l represent the average cycle time of each shuttle and lifter, $\frac{N_{S,T}}{T_s}$ and $\frac{N_{S,A}}{T_l}$ represents the throughput of all shuttles and lifters.

Consider the assumption of a dual-command cycle with seven steps, a random storage and retrieval policy, and an acceleration/deceleration effect. T_s can be calculated as follows:

$$T_s = \frac{2}{N_{S,C}} \times \sum_{i=1}^{N_{S,C}} T_s(i) + \frac{1}{N_{S,C^2} - N_{S,C}} \times \sum_{i=1}^{N_{S,C}} \sum_{\substack{j=1 \\ j\neq i}}^{N_{S,C}} T_s(i,j) + 4 \times t_s \tag{14}$$

where $T_s\,(i)$ and $T_s\,(i,j)$ represent the travel time and can be calculated as follows:

$$T_s(i) = \begin{cases} 2 \times \frac{v_s}{a_s} + \frac{i \times u_{S,w} - \frac{{v_s}^2}{a_s}}{v_s}, & i \times u_{S,w} > \frac{{v_s}^2}{a_s} \\ 2 \times \sqrt{\frac{i \times u_{S,w}}{a_s}}, & i \times u_{S,w} \leq \frac{{v_s}^2}{a_s} \end{cases} \tag{15}$$

$$T_s(i,j) = \begin{cases} 2 \times \frac{v_s}{a_s} + \frac{|i-j| \times u_{S,w} - \frac{{v_s}^2}{a_s}}{v_s}, & |i-j| \times u_{S,w} > \frac{{v_s}^2}{a_s} \\ 2 \times \sqrt{\frac{|i-j| \times u_{S,w}}{a_s}}, & |i-j| \times u_{S,w} \leq \frac{{v_s}^2}{a_s} \end{cases} \tag{16}$$

The POSC assumption states that the device remains at the task completion point when the transaction is completed. The output lift is used to transport the tote to the I/O point in the first tier. Therefore, the output lifter remains at the I/O point in the first tier. The input lifter performs the storage task, transporting the totes that need to be stored in the target tier so that the input lifter stays at the target tier associated with the storage transaction. The movement of the output lift consists of four steps.

1) travel from the first tier to the destination tier;
2) pick up the tote;
3) return to the first tier;
4) release the tote.

The input lifter:

1) moves to the first tier;
2) picks up the tote;
3) travel to destination tier;
4) release the tote.

When the destination tier is the first tier, the lifter only provides the load and unloads the tote 'smovement. Therefore, T_l was calculated as follows:

$$T_l = \frac{1}{N_{S,T}} \times \sum_{i=1}^{N_{S,T}} 2 \times T_l\,(i) + 2 \times t_l \tag{17}$$

where $T_l\,(i)$ represents the travel time and is divided into two formulations, depending on whether the peak velocity is reached.

$$T_l\,(i) = \begin{cases} 2 \times \frac{v_l}{a_l} + \frac{(i-1) \times u_{S,w} - \frac{v_l^2}{a_l}}{v_l}, & (i-1) \times u_{S,w} > \frac{v_l^2}{a_l} \\ 2 \times \sqrt{\frac{(i-1) \times u_{S,w}}{a_l}}, & (i-1) \times u_{S,w} \leq \frac{v_l^2}{a_l} \end{cases} \tag{18}$$

IV. REPLENISHMENT STRATEGY

A. REPLENISHMENT TRANSACTION

The deployment of models is an important technical solution. The method of associating the real job order with the computer algorithm language and realizing the deployment of the algorithm to the actual operating system is very important for the realization and effective display of the algorithm. Therefore, we refer to the model deployment approach in You [27], [28]. We defined the order matrix to describe the order structure according to Ma [24]. All orders in one wave are combined into an order matrix, where the rows of the order matrix represent orders, and the columns represent SKUs. We divided the orders into $I \times J$ grids, where I indicates the total number of orders in one wave and J indicates the total number of SKU. Therefore, the order matrix is defined as follows:.

$$O = (q_{ij})_{I\times J} \tag{19}$$

where, $i \in \{1, 2, \cdots, I$ represents the ith order $j \in \{1, 2, \cdots, J$ represents the jth SKU q_{ij} represents the requested quantity of the jth SKU in the ith order

The total request quantity of the jth SKU in this wave is:

$$Q_j = \sum_{i=1}^{I} q_{ij} \tag{20}$$

The request frequency matrix is defined by the following:

$$A = (a_{ij})_{I\times J} \tag{21}$$

where, $i \in \{1, 2, \cdots, I$ represents the ith order $j \in \{1, 2, \cdots, J$ represents the jth SKU a_{ij} represents the requested frequency of the jth SKU in the ith order

$$a_{ij} = \begin{cases} 1 & q_{ij} > 0 \\ 0 & q_{ij} = 0 \end{cases} \tag{22}$$

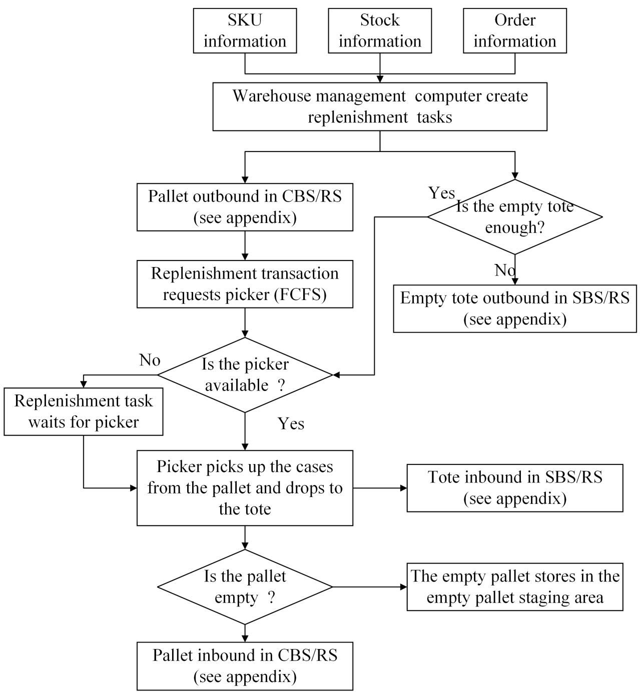


FIGURE 2. **Flowchart of the replenishment transaction in the CAWS.**

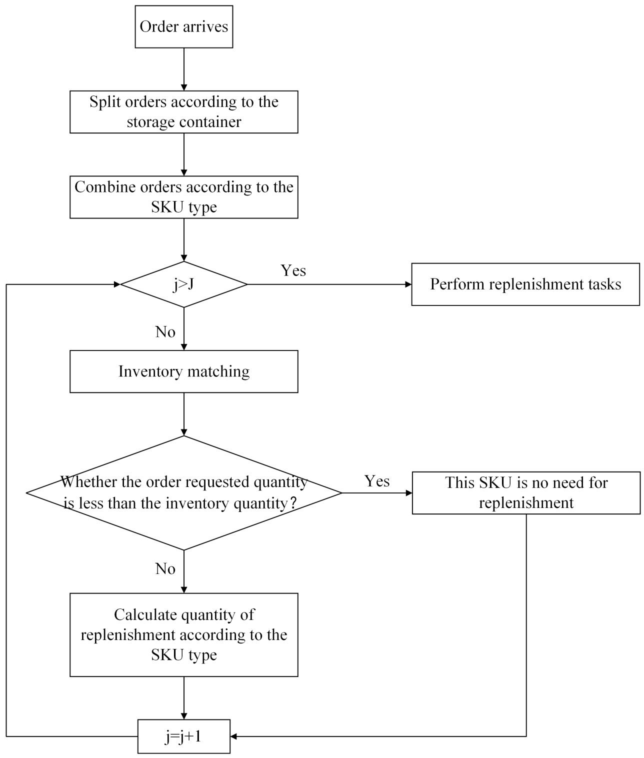


FIGURE 3. **Replenishment flow chart.**

Therefore, the total request frequency of the jth SKU in this wave is:

$$A_j = \sum_{i=1}^{I} a_{ij} \tag{23}$$

The following is several indicators to reflect the order structure:

- The size of wave α represents the total number of orders and equals to I;
- The density of order β represents the number range of the request SKU in one order and is calculated by

$$A_i = \sum_{j=1}^{J} a_{ij} \tag{24}$$

- The strength of order γ represents the request quantity range of one SKU in each order and is equal to q_{ij}.

Then the order matrix can be represented by the above three indicators:

$$O = f\left(\alpha, \beta, \gamma\right) \tag{25}$$

B. REPLENISHMENT STRATEGY

In CAWS, SBS/RS provides the cases and item selection. SBS/RS is more flexible and efficient in item-picking operations than CBS/RS, which only provides pallet picking. However, SBS/RS has limitations in terms of the amount of storage, and it is impractical to store all SKUs in SBS/RS. However, if only the requested SKUs are allowed to be stored in the SBS/RS, this inevitably increases the workload of the equipment (e.g., crane, shuttle, RGV, and lifter) in the wave cycle. Therefore, the purpose of this study is to find a balance between workload and inventory status, and find a better strategy for replenishment. This strategy can reduce the equipment workload and improve system efficiency by considering the storage positions in the SBS/RS. The replenishment flow of the CAWS is shown in Fig 3. The warehouse management computer creates replenishment tasks according to the following four steps.

Step 1: Processing When a wave of orders arrives, the computer splits the orders, labeling the entire pallet picking as the CBS/RS, labeling cases, and item picking as the SBS/RS. Subsequently, the picking of cases and items is combined according to the SKUs type. The total picking number of SKUs of each type was calculated using Equation (20).

Step 2: Inventory Matching. Similar to the form of the order matrix, we define the inventory matrix as

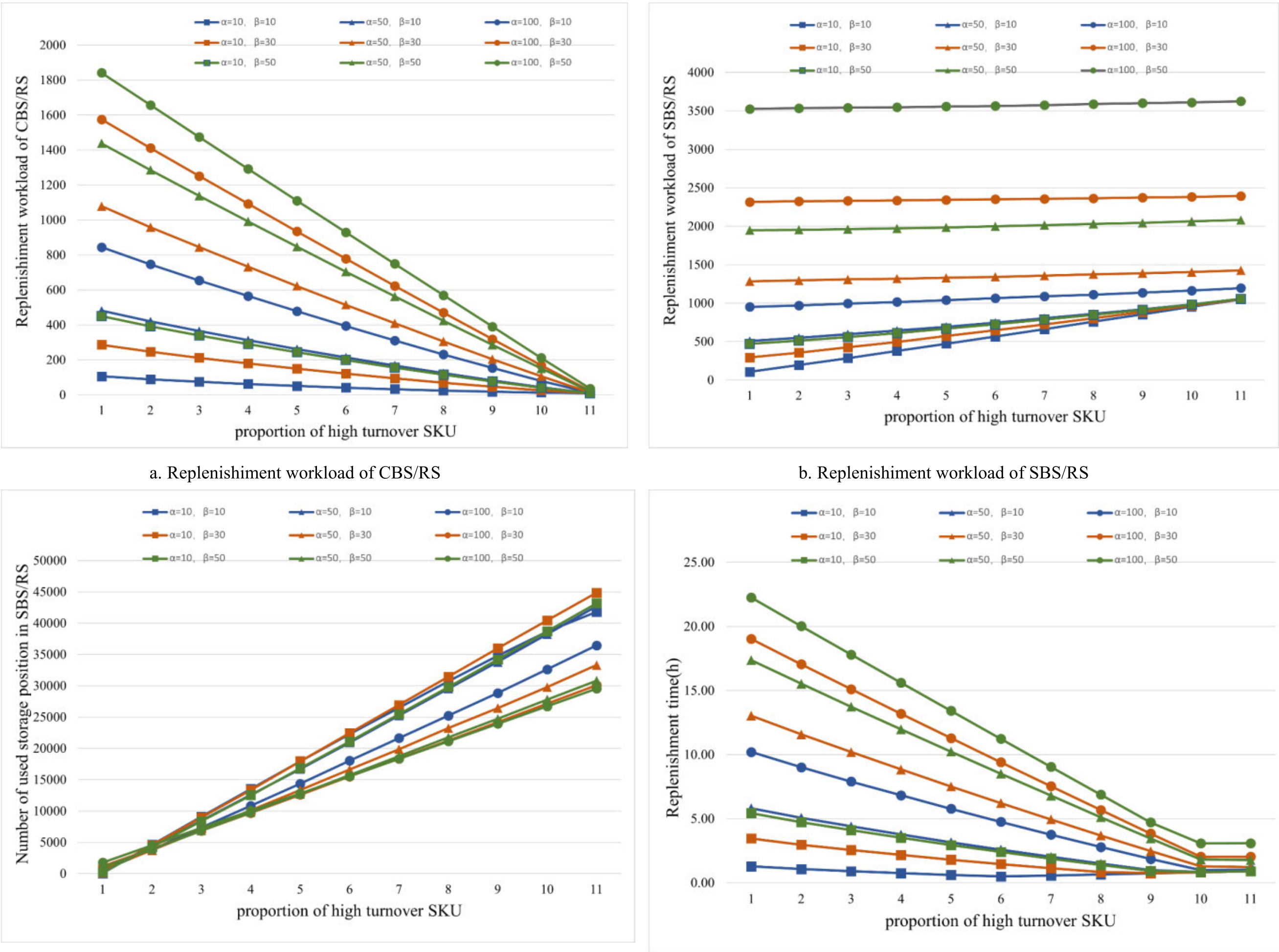
$$H = \left[h_1, h_2, \cdots, h_j, \cdots, h_J\right] \tag{26}$$

where $j \in \{1, 2, \cdots, J$ represents the jth SKU and h_j represents the stock quantity of the jth SKU.

The number of occupied storage positions can be calculated by:

$$L_o = \sum_{j=1}^{J} \left\lceil \frac{h_j}{Q_{t,j}} \right\rceil \tag{27}$$

Step 3: SKU main data matching. The SKU main date includes the full quantity of a tote, full quantity of a pallet, and



a. Replenishiment workload of CBS/RS

b. Replenishiment workload of SBS/RS

c. Number of used storage position

d. Replenishiment time(h)

FIGURE 4. Effect of order strength $\gamma = 1$.

SKU type. The SKU type is divided according to the turnover rate for a while (e.g., one year, one month, and one quarter). The SKU-type matrix is defined as follows.

$$S = \left[s_1, s_2, \cdots, s_j, \cdots, s_J\right] \tag{28}$$

where $j \in \{1, 2, \cdots, J$ represents the jth SKU, s_j represents whether the jth SKU belongs to the high-turnover type.

$$s_j = \begin{cases} 1 & \text{jth SKU belongs to the high turnover type} \\ 0 & \text{jth SKU belongs to the low turnover type} \end{cases} \tag{29}$$

Therefore, the proportion of high turnover type is calculated by:

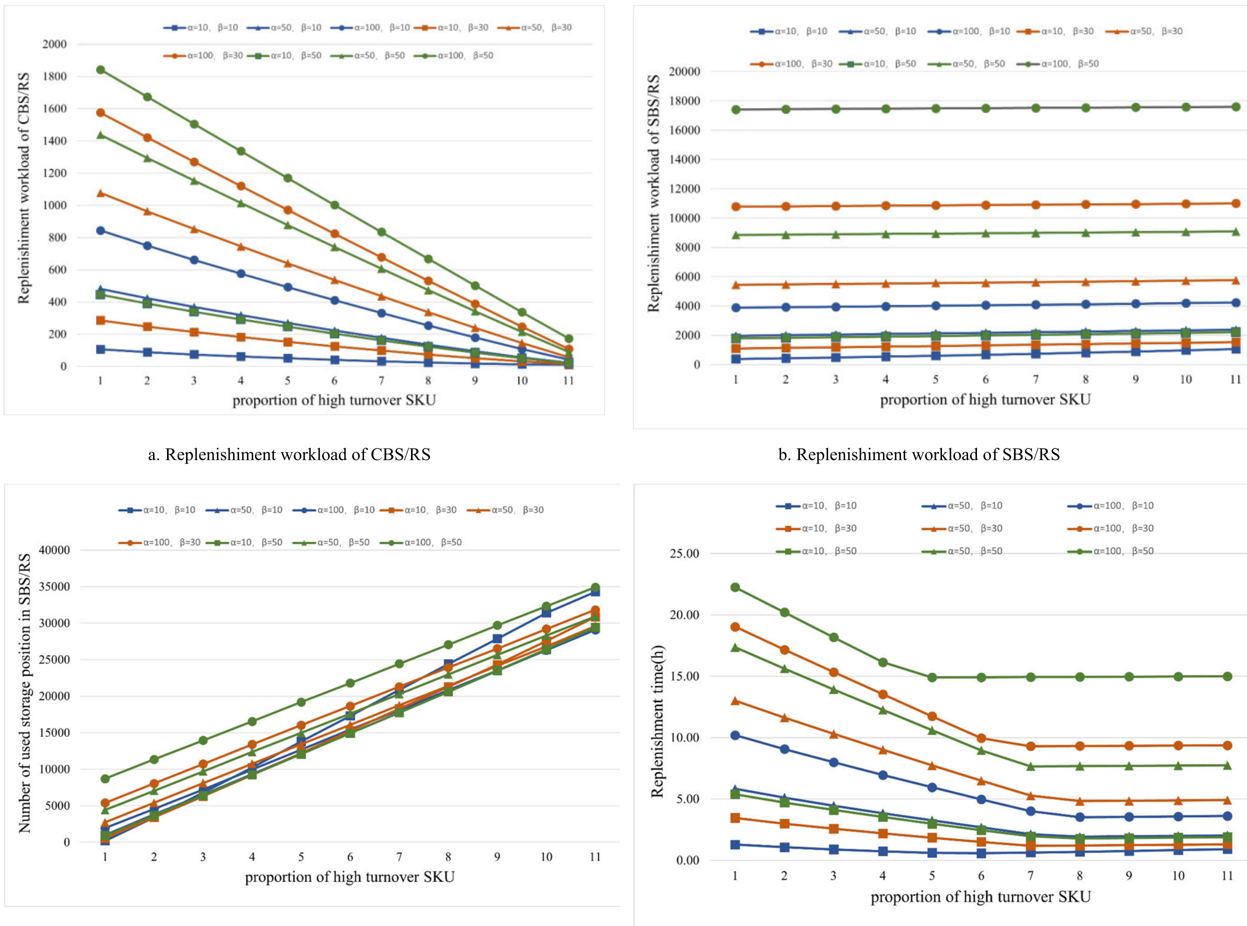
$$\rho = \frac{\sum_{j=1}^{J} S_j}{J} \tag{30}$$

When $s_j = 1$, jth SKU belongs to the high-turnover type and should be replenished excessively. $s_j = 0$, jth SKU belongs to the low-turnover type and should be replenished according to the demand. The quantity of replenishment is defined as:

$$Q_{r,j} = \begin{cases} \left\lceil \frac{\max\left(\sum_{i=1}^{I} q_{(i,j)} - h_j, 0\right)}{Q_{p,j}} \right\rceil \times Q_{p,j} s_j = 1 \\ \max\left(\sum_{i=1}^{I} q_{(i,j)} - h_j, 0\right) s_j = 0 \end{cases} \tag{31}$$

Step 4: Replenishment task. The warehouse management computer creates replenishment tasks based on the quantity of replenishment, $Q_{r,j}$. In CAWS, the replenishment task includes pallet retrieval and storage transactions in CBS/RS, empty tote retrieval transactions, and full tote storage transactions in SBS/RS, as illustrated in Fig. 2. Hence, the workload for replenishment (i.e., the number of replenishment tasks) can be calculated as follows:

$$\sum_{j=1}^{J} D_j = \sum_{j=1}^{J} D_{C,j} + D_{s,j} \tag{32}$$



a. Replenishment workload of CBS/RS

b. Replenishment workload of SBS/RS

c. Number of used storage position

d. Replenishment time(h)

FIGURE 5. Effect of order strength $\gamma = U[1, 10]$.

$D_{C,j}$ and $D_{s,j}$ represent the workload of replenishment in the CBS/RS and SBS/RS, respectively, and can be calculated using equations (33) and (34), as shown at the bottom of the next page.

C. MODELLING REPLENISHMENT

When processing each wave of orders, CAWS first handles the replenishment task and then processes the picking task. Therefore, the time the system processes a wave order is the sum of the replenishment and picking times. The processing time of one wave order can be shortened by reducing the replenishment operation time, assuming that the picking time of the order is constant. Therefore, it is evident that the optimal replenishment operation goal is the shortest replenishment time.

$$\min(\max(\sum_{j=1}^{J}\frac{D_{C,j}}{TH_C}, \sum_{j=1}^{J}\frac{D_{s,j}}{TH_S})) \tag{35}$$

Constraints:

$$\sum_{j=1}^{J} D_{C,j} < T \times TH_C \tag{36}$$

$$\sum_{j=1}^{J} D_{S,j} < T \times TH_S \tag{37}$$

$$\sum_{j=1}^{J}\left\lceil\frac{Q_{r,j}+h_j}{Q_{t,j}}\right\rceil < 2 \times N_{S,A} \times N_{S,T} \times N_{S,C} \tag{38}$$

$$Q_{t,j} < Q_{p,j} \tag{39}$$

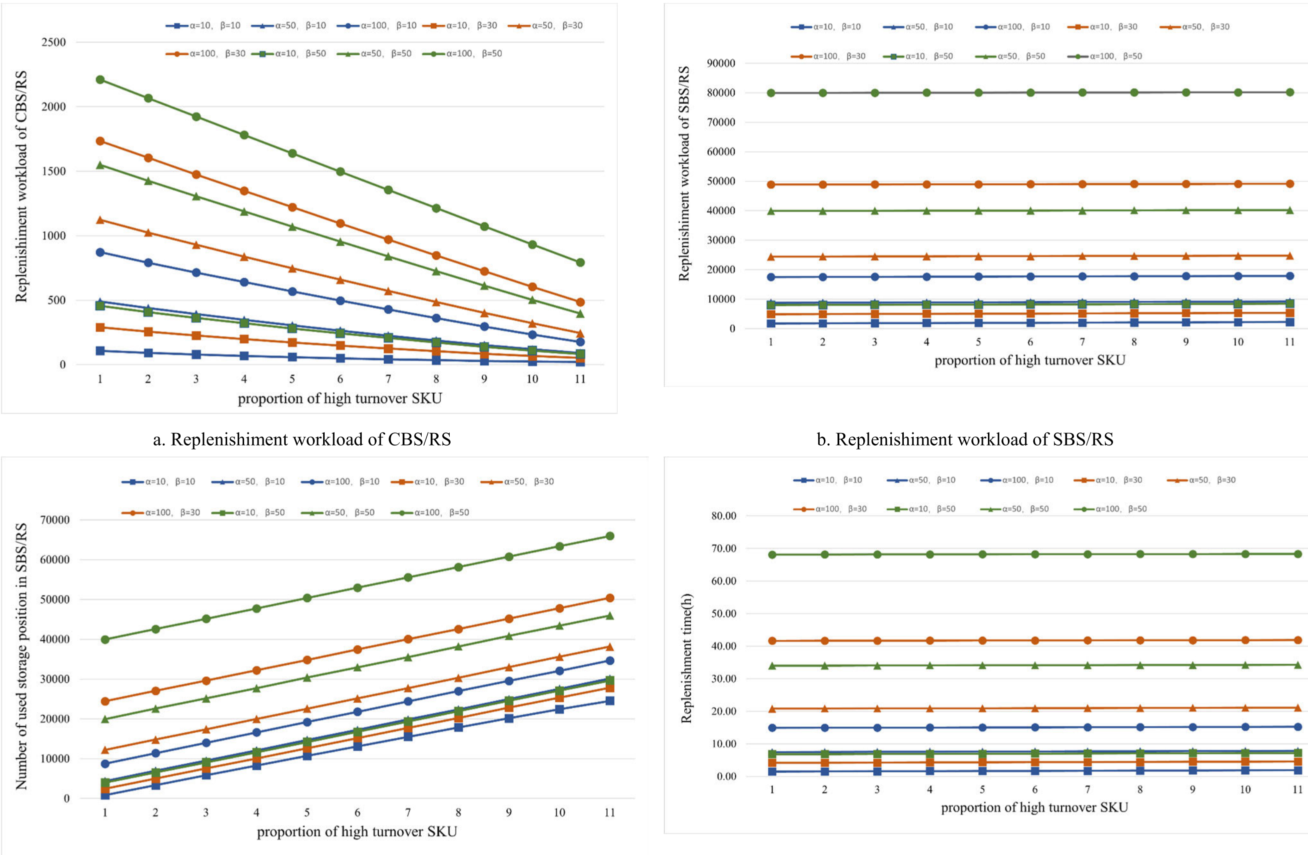
$$\sum_{j=1}^{J} s_j \leq J s_j = \{0, 1\} \tag{40}$$

Equation (35) defines the objective of minimizing the replenishment time. Constraints (36) and (37) ensure that there is sufficient equipment capacity to fulfil those replenishment tasks. Constraint (38) limits the storage positions, and constraint (39) states that the quantity of full totes is less than that of the full pallets. Constraint (40) illustrates the relationship between the SKU-type matrix and order matrix.

V. EXPERIMENTAL STUDY

A. EXPERIMENTAL DATA

Equation (25) illustrates that the order matrix includes three main factors: the size of the wave α, the density of order β and the strength of order γ. Considering order structure



a. Replenishment workload of CBS/RS

b. Replenishment workload of SBS/RS

c. Number of used storage position

d. Replenishment time(h)

FIGURE 6. Effect of order strength $\gamma = U[1, 50]$.

suitability, we compared the workload of CBS/RS and SBS/RS, the number of used storage positions, and the replenishment time. Table 1 lists the different order structure factors (γ, β, α) and the proportion of high-turnover SKU ρ.

Order structure has a greater impact on the formulation of replenishment strategies (for example, what is the proportion of high-turnover SKU How much to set the size of a wave), which directly affects the operational efficiency of

$$D_{C,j} = \begin{cases} \left\lceil \dfrac{\max\left(\sum_{i=1}^{I} q_{(i,j)} - h_j, 0\right)}{Q_{p,j}} \right\rceil s_j = 1 \\ 1 + \left\lfloor \dfrac{\sum_{i=1}^{I} q_{(i,j)} - h_j}{Q_{p,j}} \right\rfloor s_j = 0, \left(\sum_{i=1}^{I} q_{(i,j)} - h_j\right) modQ_{p,j} > 0 \\ \dfrac{\sum_{i=1}^{I} q_{(i,j)} - h_j}{Q_{p,j}} s_j = 0, \left(\sum_{i=1}^{I} q_{(i,j)} - h_j\right) modQ_{p,j} = 0 \\ 0 s_j = 0, \left(\sum_{i=1}^{I} q_{(i,j)} - h_j\right) modQ_{p,j} < 0 \end{cases} \tag{33}$$

$$D_{S,j} = \begin{cases} 2 \times \left\lceil \left\lceil \dfrac{\max\left(\sum_{i=1}^{I} q_{(i,j)} - h_j, 0\right)}{Q_{p,j}} \right\rceil \times \dfrac{Q_{p,j}}{Q_{t,j}} \right\rceil s_j = 1 \\ 2 \times \left\lceil \dfrac{\max\left(\sum_{i=1}^{I} q_{(i,j)} - h_j, 0\right)}{Q_{p,j}} \right\rceil s_j = 0 \end{cases} \tag{34}$$

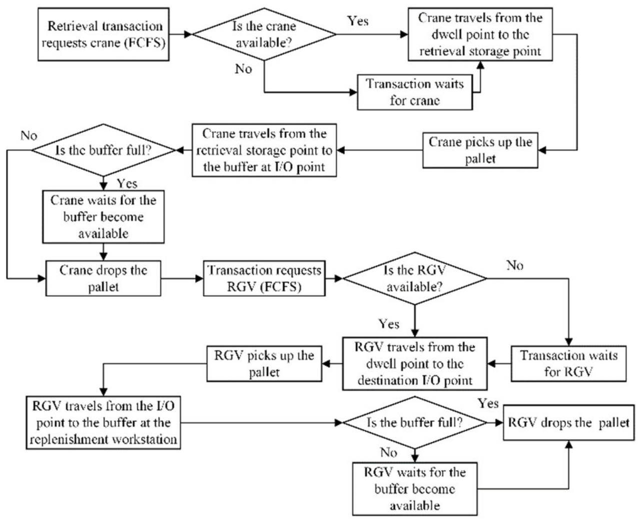


FIGURE 7. Pallet outbound in CBS/RS.

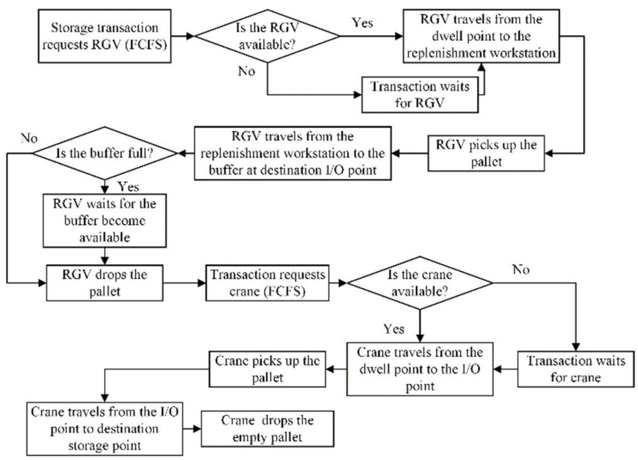


FIGURE 8. Pallet inbound in CBS/RS.

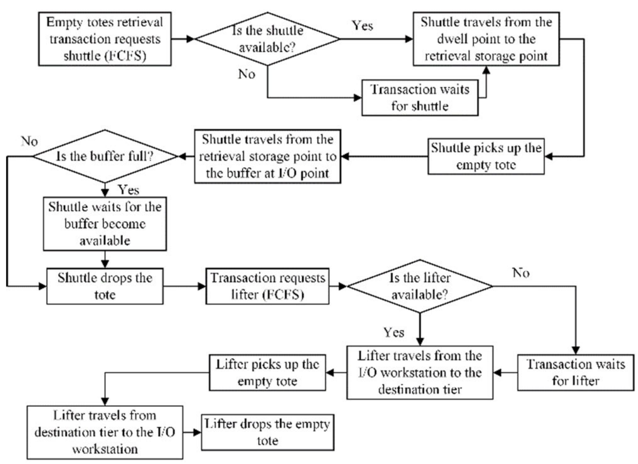


FIGURE 9. Empty tote outbound in SBS/RS.

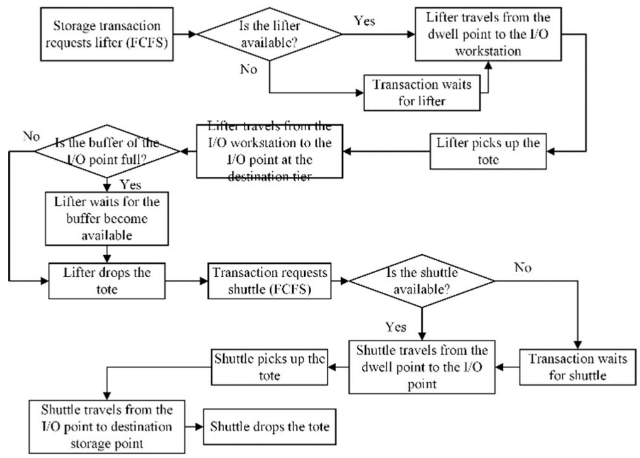


FIGURE 10. Tote inbound in SBS/RS.

TABLE 1. Experimental parameters.

γ	β	α	ρ
1	10:10:100	10:10:100	1:10%:100%
1~10	10:10:100	10:10:100	1:10%:100%
1~50	10:10:100	10:10:100	1:10%:100%

TABLE 2. Experimental parameters of caws.

Parameters	Notation	Value
Number of CBS/RS storage aisles, tiers and columns	$N_{C,A}, N_{C,T}, N_{C,C}$	2,5,60
Number of SBS/RS storage aisles, tiers and columns	$N_{S,A}, N_{S,T}, N_{S,C}$	1,15,140
Unit width, depth and height per storage position of SBS/RS(m)	$u_{S,w}, u_{S,d}, u_{S,h}$	0.5,0.7,0.5
Unit width, depth and height per storage position of CBS/RS(m)	$u_{C,w}, u_{C,d}, u_{C,h}$	1.16,1.2,1.7
Vertical maximum velocity and horizontal maximum velocity of crane(m/s)	vv_{cr}, hv_{cr}	1,3
Vertical and horizontal acceleration/deceleration of crane(m/s^2)	va_{cr}, ha_{cr}	0.5,0.6
Maximum Velocity of RGV, shuttle, lifter(m/s)	v_R, v_s, v_l	2,3,3
Acceleration/ deceleration rate of l RGV, shuttle, lifter(m/s^2)	a_R, a_s, a_l	1.5,1,3
Number of RGVs, I/O points and I/O workstations in the CBS/RS	$N_{C,R}, N_{C,I,P}, N_{C,I,W}$	1,2,2
Fixed time required for the crane, lift, shuttle and RGV to load or unload the tote	t_{cr}, t_l, t_s, t_R	10,5,4,1

the distribution center. Without loss of generality, we set ten groups of experiments for each scenario, with each group containing 100 waves. The strength of order γ represents the requested quantity of SKU in each order. We set a fixed value of 1, uniform distribution U [1], [10], and U[1,50]. The system parameters are listed in Table 2.

The effect of order strength γ is shown in Fig 4 $\sim$ 6. The following observations on the replenishment transaction of CAWS can be made based on these results.

1) For the replenishment workload of CBS/RS, when the proportion of SKUs with high turnover frequency increases, the total number of tasks of the CBS/RS decreases, and when the size of the wave decreases, the total number of tasks of the CBS/RS decreases. When the order density decreased, the total number of tasks in the CBS/RS decreased. When the order intensity decreased, the total number of tasks in the CBS/RS decreased. Therefore, when the order intensity and density are large, the total workload in the CBS/RS can be reduced by reducing the wave size and increasing the proportion of SKUs with a high turnover frequency.
2) Replenishment workload of SBS/RS: When the proportion of SKUs with high turnover frequency decreases, the replenishment workload of SBS/RS decreases. When the size of the wave decreased, the replenishment workload of the SBS/RS decreased. When the order density decreased, the replenishment workload of SBS/RS decreased. When the order intensity decreased, the replenishment workload of SBS/RS decreased. Therefore, when the order intensity and order density are large, the replenishment workload of SBS/RS can be reduced by reducing the wave size and proportion of SKUs with high turnover frequency.
3) For the number of used storage positions, when the proportion of SKUs with high turnover frequency increased, the storage position occupied in the SBS/RS increased. When the wave size increased, the storage position occupied by the SBS/RS increased. When the order density increased, the storage position occupied by the SBS/RS increased. When the order strength increased, the storage position occupied by the SBS/RS increased. Therefore, if the free storage position is insufficient, the storage position occupied in the SBS/RS can be reduced by reducing the wave size and proportion of SKUs with a high turnover frequency.
4) Replenishment time: When the proportion of SKUs with high turnover frequency increases, the system replenishment operation time decreases. When the wave size decreases, the system replenishment operation time decreases. When the order density decreases, the system replenishment operation time decreases, and when the order strength is reduced, the system replenishment operation time is reduced. Therefore, the system replenishment operation time can be shortened by appropriately increasing the proportion of SKUs with a high turnover frequency and by reducing the wave size.

B. EXAMPLE VERIFICATION

The application scenario is the distribution center of the supply distributor and e-commerce platform customer of a daily production enterprise. The largest four days of order were selected, as shown in Table 3. To avoid the single nature of the order structure, we select non-continuous 4 waves of orders to test each day. α is the size of each wave, γ_c is the average order strength for the cases picking, ρ_{opt} is the best high turnover type SKU ratio under the current order structure combination Fig 4 $\sim$ Fig 6, T_{rot} is the replenishment operation time. $\rho = 0$ means that all SKUs are set to the low turnover type (i.e., each wave in the replenishment strategy is replenished according to the order). $\rho = 1$ means that all SKUs are set to a high-turnover type (i.e., each wave of replenishment strategies is replenished according to the pallet capacity per wave). To ensure consistency of the comparison test, the initial inventory sets the same data for each experiment. When the value of ρ changes, the inventory of the SBS/RS returns to the initial inventory state. Table 3 shows the appropriate proportion of high-turnover SKU ratios, which can effectively shorten the replenishment operation time.

TABLE 3. CAWS replenishment operation time(h).

Day	α	γ_c	ρ_{opt}	T_{rot}		
				$\rho = 0$	$\rho = 1$	$\rho = \rho_{opt}$
1	200	3	90.05%	0.62	0.30	0.28
	152	8	40.04%	0.53	0.48	0.41
	79	7	38.56%	0.32	0.28	0.24
	114	6	54.24%	0.42	0.31	0.30
2	189	4	81.78%	0.61	0.33	0.32
	300	4	80.99%	0.76	0.46	0.45
	30	3	31.30%	0.14	0.22	0.13
	20	7	18.99%	0.10	0.22	0.09
3	232	3	93.37%	0.67	0.31	0.31
	200	8	39.37%	0.62	0.58	0.52
	110	1	69.29%	0.40	0.25	0.24
	20	2	23.49%	0.09	0.22	0.09
4	77	6	44.47%	0.31	0.27	0.23
	30	2	29.52%	0.13	0.23	0.11
	411	3	66.74%	0.86	0.49	0.47
	200	2	95.53%	0.63	0.25	0.23

VI. CONCLUSION

This study introduces a new automatic warehouse-sorting system (CAWS). This system has the intensive storage capacity of CBS/RS and combines the flexibility and high throughput of SBS/RS. This study focuses on the replenishment strategy of the system and studies how to optimize the replenishment operation of the system by adjusting the wave size and the proportion of SKUs with high turnover frequency under the premise of considering the order structure.

irst, a throughput model was established by analyzing the workflow and system structure. The order matrix is then established to study the effect of the order structure on the replenishment strategy. In the experimental study, we analyzed the effects of order size and order density on relevant parameters in the replenishment strategy, such as the wave size and proportion of high turnover SKU. This study provides a real case study to verify the effectiveness of the replenishment strategy. Warehouse operation managers can increase the picking throughput of the system by reducing the wave size, increasing the proportion of high-turnover SKUs, and minimizing replenishment time.

In addition, the order structure can be explored in the future. Some rules and indicators can be described by order density, order strength, order size, and other parameters to guide the determination of the relevant parameters of the replenishment strategy.

APPENDIX A

When the replenishment transaction arrives, the CBS/RS processes the pallet outbound and pallet inbound, whereas the SBS/RS processes the empty tote outbound and tote inbound. A flowchart is shown in Fig 7–10.

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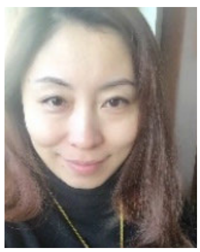

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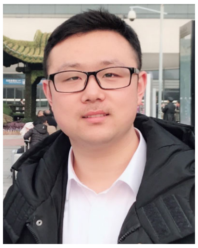

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